1845 Trillium flore pedunculato cernuo, Lin. Sp. Pl. 339.

1846 Valeriana montana ſubrotundo folio, C. B. P. 165.

1847 Vicia pedunculis multifloris, petiolis polyphillis, foliolis lanceolatis glabris, Hort. Upſal, 219.

1848 Vicia ſylvatica multiflora maxima perennis, tetro odore, floribus albentibus, lineis cæruleis ſtriatis, Pluk. Alm. 387.

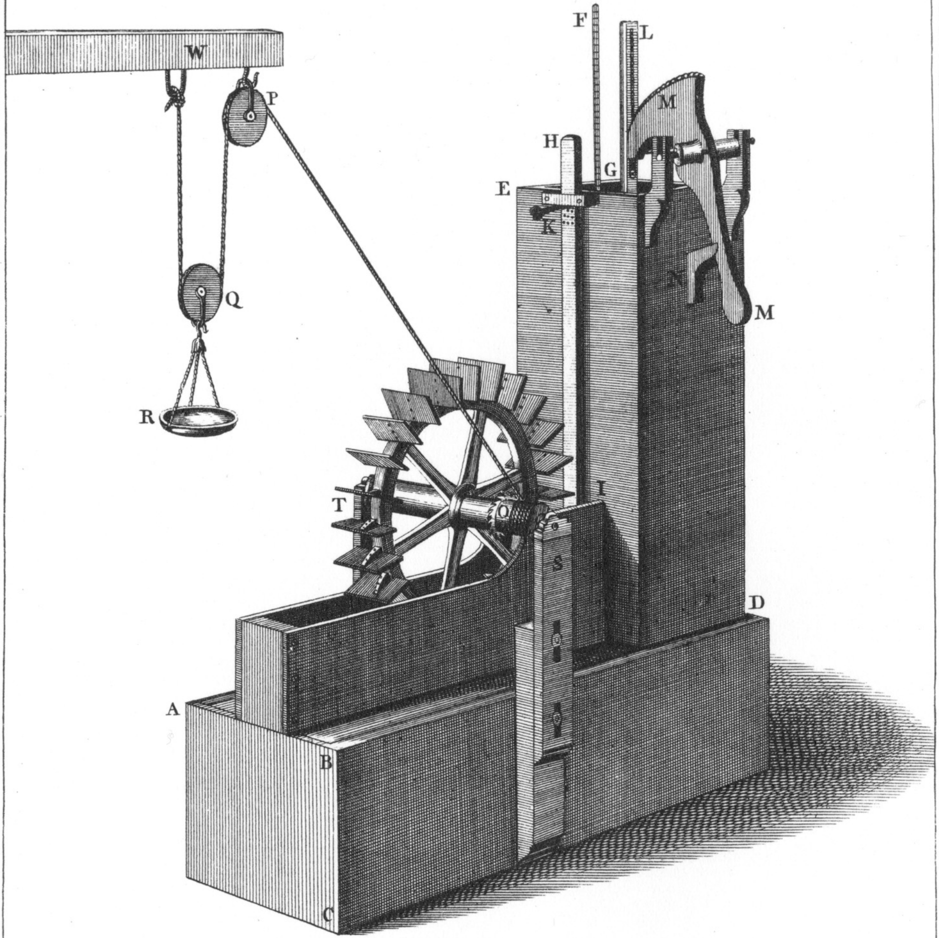
1849 Vinca foliis oblongo-ovatis integerrimis, tubo floris longiſſimo, caule ramoſo fruticoſo, Miller's Iconſ.

1850 Xanthium, ſive Lappa minor, J. B. 3. 572.
Lappa minor, ſive Xanthium Dioſcorid. C. B. P. 198.

XVIII. *An experimental Enquiry concerning the natural Powers of Water and Wind to turn Mills, and other Machines, depending on a circular Motion. By Mr.* J. Smeaton, *F. R. S.*

Read May 3, & 10, 1759.

WHAT I have to communicate on this ſubject was originally deduced from experiments made on working models, which I look upon as the beſt means of obtaining the outlines in mechanical enquiries. But in this caſe it is very neceſſary to diſtinguiſh the circumſtances in which a model differs from a machine in large; otherwiſe a model is more apt to lead us from the truth



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truth than towards it. Hence the common obſervation, that a thing may do very well in a model, that will not anſwer in large. And indeed, tho' the utmoſt circumſpection be uſed in this way, the beſt ſtructure of machines cannot be fully aſcertained, but by making trials with them, when made of their proper ſize. It is for this reaſon, that, tho' the models referred to, and the greateſt part of the following experiments, were made in the years 1752 and 1753, yet I deferred offering them to the Society, till I had an opportunity of putting the deductions made therefrom in real practice, in a variety of caſes, and for various purpoſes; ſo as to be able to aſſure the Society, that I have found them to anſwer.

PART I.

Concerning UNDERSHOT WATER-WHEELS.

PLATE IV. Fig. 1. is a perſpective view of the machine for experiments on water-wheels; wherein

ABCD is the lower ciſtern, or magazine, for receiving the water, after it has quitted the wheel; and for ſupplying

DE the upper ciſtern, or head; wherein the water being raiſed to any height required, by a pump, that height is ſhewn by

F G, a ſmall rod, divided into inches and parts; with a float at the bottom, to move the rod up and down, as the ſurface of the water riſes and falls.

H I is a rod by which the ſluice is drawn, and ſtopt at any height required, by means of

K a pin or peg, which fits ſeveral holes, placed

in the manner of a diagonal ſcale, upon the face of the rod H I.

G L is the upper part of the rod of the pump, for drawing the water out of the lower ciſtern, in order to raiſe and keep up the ſurface thereof at its deſired height, in the head D E; thereby to ſupply the water, expended by the aperture of the ſluice.

M M is the arch and handle for working the pump, which is limited in its ſtroke by

N a piece for ſtopping the handle from raiſing the piſton too high; that alſo being prevented from going too low, by meeting the bottom of the barrel.

O is the cylinder, upon which a cord winds, and which being conducted over the pullies P and Q, raiſes

R, the ſcale, into which the weights are put, for trying the power of the water.

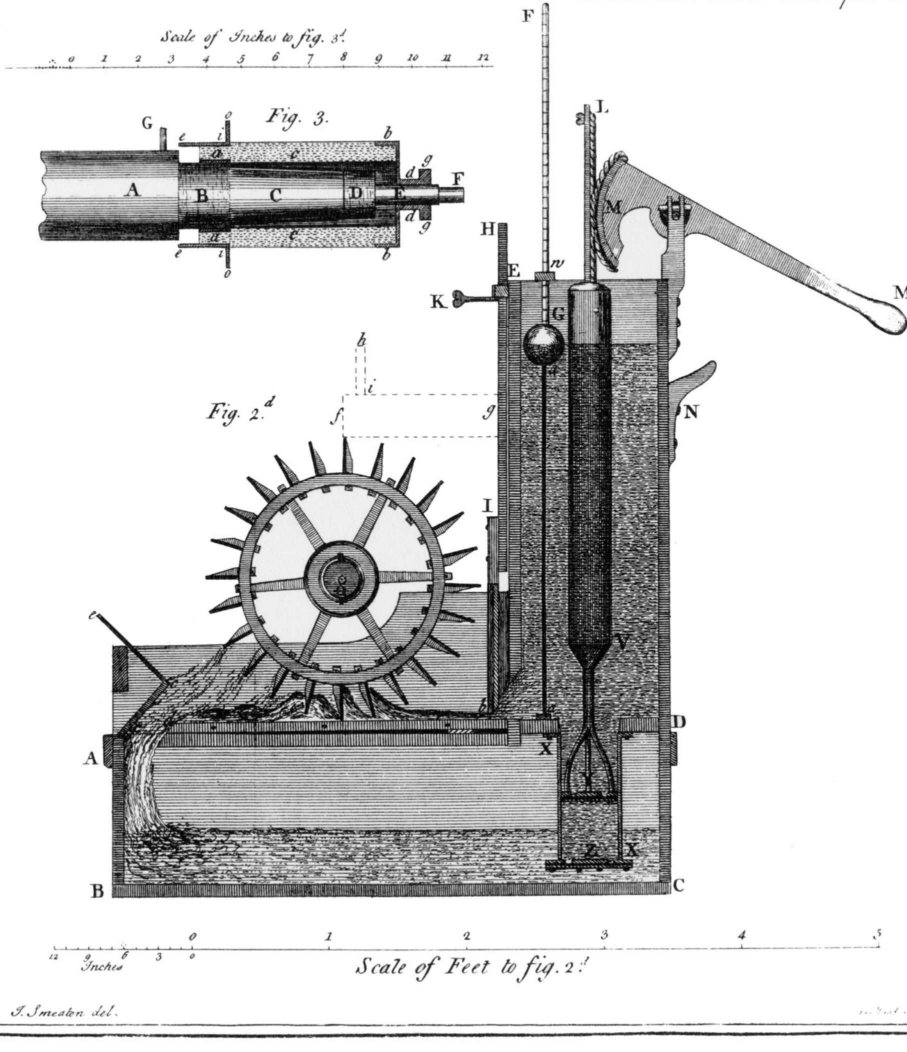
S T the two ſtandards, which ſupport the wheel, are made to ſlide up and down, in order to adjuſt the wheel, as near as poſſible, to the floor of the conduit.

W the beam which ſupports the ſcale and pullies; this is repreſented as but little higher than the machine, for the ſake of bringing the figure into a moderate compaſs, but in reality is placed 15 or 16 feet higher than the wheel.

Plate V. Fig. 2. is a ſection of the ſame machine, wherein the ſame parts are marked with the ſame letters as in Fig. 1. Beſides which

X X is the pump barrel, being 5 inches diameter, and 11 inches long.

Y is



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Y is the piſton; and

Z the fixed valve.

G V is a cylinder of wood, fixed upon the pump-rod, and reaches above the ſurface of the water; this piece of wood being of ſuch a thickneſs, that its ſection is half the area of that of the pump-barrel, will cauſe the ſurface of water to riſe in the head, as much while the piſton is deſcending, as while it is riſing: and will thereby keep the gauge-rod F G more equally to its height. *Note*, the arch and handle M M is here repreſented on a different ſide to what it is ſhewn in the preceding figures, in order that its dimenſions may the better appear.

a a ſhews one of the two wires which ſerve as directors to the float, in order that the gauge rod F G may be kept perpendicular; for the ſame purpoſe alſo ſerves *w*, a piece of wood with a hole to receive the gauge-rod, and keep it upright.

b is the aperture of the ſluice.

c c a kant-board, for throwing the water more directly down the opening *c d*, into the lower ciſtern: and

c e is a ſloping board, for bringing back the water that is thrown up by the floats of the wheel.

Fig. 3. repreſents one end of the main axis, with a ſection of the moveable cylinder, marked O in the preceding figures.

A B C D is the end of the axis; whereof the parts

B and D are covered with ferrules or hoops of braſs.

E is a cylinder of metal; whereof the part marked

F is

F is the pivot or gudgeon.

c c is the ſection of an hollow cylinder of wood, the diameter of the interior part being ſomewhat larger than the cylindrical ferrule B.

a a is the ſection of a ferrule of braſs, driven into the end of the hollow cylinder, and which is adjuſted to that marked B, ſo as to ſlide freely thereupon, but with as little ſhake as poſſible.

b b, *d d*, *g g*, repreſent the ſection of a braſs ferrule, plate, and ſocket, fixed upon the other end of the hollow cylinder; the ſocket *d d* being adjuſted to ſlide freely upon the cylinder E, in the ſame manner as the ferrule *a a* ſlides upon the cylinder B: the outer end of the ſocket at

g g is formed into a ſort of button; by puſhing whereof, the hollow cylinder will move backwards and forwards, or turn round at pleaſure upon the cylindrical parts of the axis B and E.

e e, *i i*, *o o*, repreſent the ſection of a braſs ferrule, alſo fixed upon the hollow cylinder: the edge of this ferrule

e e is cut into teeth, in the manner of a *contrate* wheel; and the edge thereof

o o is cut in the manner of a ratchett.

Of conſequence, when the plate *b d d b* is puſhed cloſe to the ferrule D, the teeth of the ferrule *e e* will lay hold of

G, a pin fixed into the axis; by which means the hollow cylinder is made to turn along with the wheel and axis: but being drawn back by the button *g g*, the hollow cylinder is thereby diſengaged from the pin G, and ceaſes turning.

Note. The weight in the ſcale is prevented from

from running back, by a catch that plays in and lays hold of the ratchet *o o*.

By this means the hollow cylinder upon which the cord winds, and raiſes the weight, is put in action and diſcharged therefrom inſtantaneouſly, while the wheel is in motion: for without ſome contrivance of this kind, it would not be eaſy to make this ſort of experiments with any tolerable degree of exactneſs.

The uſe of the apparatus now deſcribed will be rendered more intelligibie, by giving a general idea of what I had in view; but as I ſhall be obliged to make uſe of a term which has heretofore been the cauſe of diſputation, I think it neceſſary to aſſign the ſenſe in which I would be uuderſtood to uſe it; and in which I apprehend it is uſed by practical *Mechanicks.*

The word *Power*, as uſed in practical mechanicks, I apprehend to ſignify the exertion of ſtrength, gravitation, impulſe, or preſſure, ſo as to produce motion: and by means of ſtrength, gravitation, impulſe, or preſſure, compounded with motion, to be capable of producing an effect: and that no effect is properly mechanical, but what requires ſuch a kind of power to produce it.

The raiſing of a weight, relative to the height to which it can be raiſed in a given time, is the moſt proper meaſure of power; or, in other words, if the weight raiſed is multiplied by the height to which it can be raiſed in a given time, the product is the meaſure of the power raiſing it; and conſequently, all thoſe powers are equal, whoſe products, made by

 ſuch

ſuch multiplication, are equal: for if a power can raiſe twice the weight to the ſame height; or the ſame weight to twice the height, in the ſame time that another power can, the firſt power is double the ſecond: and if a power can raiſe half the weight to double the height; or double the weight to half the height, in the ſame time that another can, thoſe two powers are equal. But note, all this is to be underſtood in caſe of ſlow or equable motion of the body raiſed; for in quick, accelerated, or retarded motions, the *vis inertiæ* of the matter moved will make a variation.

In comparing the effects produced by water-wheels, with the powers producing them; or, in other words, to know what part of the original power is neceſſarily loſt in the application, we muſt previouſly know how much of the power is ſpent in overcoming the friction of the machinery, and the reſiſtance of the air; alſo what is the real velocity of the water at the inſtant that it ſtrikes the wheel; and the real quantity of water expended in a given time.

From the velocity of the water, at the inſtant that it ſtrikes the wheel, given; the height of head productive of ſuch velocity can be deduced, from acknowleged and experimented principles of hydroſtatics: ſo that by multiplying the quantity, or weight of water, really expended in a given time, by the height of head ſo obtained; which muſt be conſidered as the height from which that weight of water had deſcended in that given time; we ſhall have a product, equal to the original power of the water; and clear of all uncertainty, that would ariſe from the friction of the water, in paſſing ſmall apertures;

and

and from all doubts, arifing from the different meafure of fpouting waters, affigned by different authors. On the other hand, the fum of the weights raifed by the action of this water, and of the weight required to overcome the friction and refiftance of the machine, multiplied by the height to which the weight can be raifed in the time given, the product will be equal to the effect of that power; and the proportion of the two products will be the proportion of the *power* to the *effect*: fo that by loading the wheel with different weights fucceffively, we fhall be able to determine at what particular load, and velocity of the wheel, the effect is a *maximum*.

The manner of finding the real velocity of the water, at the inftant of its ftriking the wheel; the manner of finding the value of the friction, refiftance, &c. in any given cafe; and the manner of finding the real expence of water, fo far as concerns the following experiments, without having recourfe to theory; being matters upon which the following determinations depend, it will be neceffary to explain them.

To determine the Velocity of the Water ftriking the Wheel.

It has already been mentioned, in the references to the figures, that weights are raifed by a cord winding round a cylindrical part of the axis. Firft, then, let the wheel be put in motion by the water, but without any weights in the fcale; and let the number of turns in a minute be 60: now it is evident, that was the wheel free from friction and refiftance, that 60 times the circumference of the wheel

would be the ſpace through which the water would have moved in a minute; with that velocity wherewith it ſtruck the wheel: but the wheel being incumbred by friction and reſiſtance, and yet moving 60 turns in a minute, it is plain, that the velocity of the water muſt have been greater than 60 circumferences before it met with the wheel. Let now the cord be wound round the cylinder, but contrary to the uſual way, and put a weight in the ſcale; the weight ſo diſpoſed (which may be called the *counter-weight)* will endeavour to aſſiſt the wheel in turning the ſame away, as it would have been turned by the water: put therefore as much weight into the ſcale as, without any water, will cauſe it to turn ſomewhat faſter than at the rate of 60 turns in a minute; ſuppoſe 63: let it now be tried again by the water, aſſiſted by the weight; the wheel therefore will now make more than 60 turns; ſuppoſe 64: hence we conclude the water ſtill exerts ſome power in giving motion to the wheel. Let the weight be again increaſed, ſo as to make 64½ turns in a minute without water: let it once more be tried with water as before; and ſuppoſe it now to make the ſame number of turns with water as without. *viz.* 64½: hence it is evident, that in this caſe the wheel makes the ſame number of turns in a minute, as it would do if the wheel had no friction or reſiſtance at all; becauſe the weight is equivalent thereto; for was it too little, the water would accelerate the wheel beyond the weight; and if too great, retard it; ſo that the water now becomes a *regulator* of the wheel's motion; and the velocity of its circumference becomes a meaſure of the velocity of the water.

In

In like manner, in ſeeking the greateſt product, or *maximum* of effect; having found by trials what weight gives the greateſt product, by ſimply multiplying the weight in the ſcale by the number of turns of the wheel, find what weight in the ſcale, when the cord is on the contrary ſide of the cylinder, will cauſe the wheel to make the ſame number of turns the ſame way, without water; it is evident that this weight will be nearly equal to all friction and reſiſtance taken together; and conſequently, that the weight *in* the ſcale, with twice * the weight *of* the ſcale, added to the back or counter-weight, will be equal to the weight that could have been raiſed, ſuppoſing the machine had been without friction or reſiſtance; and which multiplied by the height to which it was raiſed, the product will be the greateſt effect of that power.

The quantity of water expended is found thus:

The pump made uſe of for repleniſhing the head with water was ſo carefully made, that no water eſcaping back by the leathers, it delivered the ſame quantity of water at every ſtroke, whether worked quick or ſlow; and as the length of the ſtroke was limited, conſequently the value of one ſtroke (or on account of more exactneſs 12 ſtrokes) was known, by the height to which the water was thereby raiſed in the head; which being of a regular figure was eaſily meaſured. The ſluice, by which the water was drawn upon the wheel, was made to ſtop at certain heights by a peg; ſo that when the peg was in the ſame hole,

* The weight of the ſcale makes part of the weight both ways.

the aperture for the effluent water was the ſame. Hence the quantity of water expended by any given head, and opening of the ſluice, may be obtained: for by obſerving how many ſtrokes a minute was ſufficient to keep up the ſurface of the water at the given height, and multiplying the number of ſtrokes by the value of each, the water expended by any given aperture and head in a given time will be given.

Theſe things will be further illuſtrated by going over the *calculus* of one ſett of experiments.

Specimen of a Sett of Experiments.

The ſluice drawn to the 1ſt hole.

The water above the floor of the ſluice		30 Inches.
Strokes of the pump in a minute	—	$39\frac{1}{2}$
The head raiſed by 12 ſtrokes	—	21 Inches.
The wheel raiſed the empty ſcale, and made turns in a minute	— —	80
With a counter-weight of 1 lb. 8 oz. it made		85
D° tried with water	— —	86

N°		Weight. lb.	oz.		Turns in a min.		Product.
1	—	4	0	—	45	—	180
2	—	5	0	—	42	—	210
3	—	6	0	—	$36\frac{1}{4}$	—	$217\frac{1}{2}$
4	—	7	0	—	$33\frac{3}{4}$	—	$236\frac{1}{4}$
5	—	8	0	—	30	—	240 maximum.
6	—	9	0	—	$26\frac{1}{2}$	—	$238\frac{1}{2}$
7	—	10	0	—	22	—	220
8	—	11	0	—	$16\frac{1}{2}$	—	$181\frac{1}{2}$
9	—	12	* ceaſed working.				

* *N. B.* When the wheel moves ſo ſlow as not to rid the water ſo faſt as ſupplied by the ſluice, the accumulated water falls back upon the aperture, and the wheel immediately ceaſes moving.

Counter

Counter-weight, for 30 turns without water, 2 oz. in the ſcale.

N. B. The area of the head was 105,8 ſquare inches.
Weight of the empty ſcale and pulley, 10 oz.
Circumference of the cylinder, 9 inches.
Circumference of the water-wheel, 75 ditto.

Reduction of the above Sett of Experiments.

The circumference of the wheel, 75 inches, multiplied by 86 turns, gives 6450 inches for the velocity of the water in a minute; $\frac{1}{60}$ of which will be the velocity in a ſecond, equal to 107,5 inches, or 8,96 feet, which is due to a head of 15 inches *; and this we call the *virtual* or *effective* head.

The area of the head being 105,8 inches, this multiplied by the weight of water of the inch cubic, equal to the decimal ,579 of the ounce avoirdupoiſe, gives 61,26 ounces for the weight of as much water, as is contained in the head, upon 1 inch in depth, $\frac{1}{16}$ of which is 3,83 pounds; this multiplied by the depth 21 inches, gives 80,43 lb. for the value of 12 ſtrokes; and by proportion, $39\frac{1}{2}$ (the number made in a minute) will give 264,7 lb. the weight of water expended in a minute.

Now as 264,7 lb. of water may be conſidered as having deſcended through a ſpace of 15 inches in a minute, the product of theſe two numbers 3970 will expreſs the *power* of the water to produce mechanical effects; which were as follows.

* This is determined upon the common maxim of hydroſtatics, that the velocity of ſpouting waters is equal to the velocity that an heavy body would acquire in falling from the height of the reſervoir; and is proved by the riſing of jets to the height of their reſervoirs nearly.

The

The velocity of the wheel at the *maximum*, as appears above, was 30 turns a minute; which multiplied by 9 inches, the circumference of the cylinder, makes 270 inches; but as the ſcale was hung by a pulley and double line, the weight was only raiſed half of this, *viz.* 135 inches.

The weight in the ſcale at the maximum	8 *lb.*	0 *oz.*
Weight of the ſcale and pulley ——	0	10
Counterweight, ſcale, and pulley ——	0	12
Sum of the reſiſtance —— ——	9	6
	or *lb.* 9,375.	

Now as 9,375 lb. is raiſed 135 inches, theſe two numbers being multiplied together, the product is 1266, which expreſſes the effect produced at a maximum: ſo that the proportion of the *power* to the *effect* is as 3970 : 1266, or as 10 : 3,18.

But tho' this is the greateſt *ſingle* effect producible from the power mentioned, by the impulſe of the water upon an underſhot wheel; yet, as the whole power of the water is not exhauſted thereby, this will not be the true ratio between the *power* of the water, and the *ſum* of all the *effects* producible therefrom: for as the water muſt neceſſarily leave the wheel with a velocity equal to the wheel's circumference, it is plain that ſome part of the power of the water muſt remain after quitting the wheel.

The velocity of the wheel at the maximum is 30 turns a minute; and conſequently its circumference moves at the rate of 3,123 feet a ſecond, which anſwers to a head 1,82 inches; this being multiplied by the expence of water in a minute, *viz.* 264,7 lb. produces 481 for the power *remaining* in the water after it has paſſed the wheel: this being therefore

deducted

deducted from the original power 3970, leaves 3489, which is that *part* of the power which is ſpent in producing the effect 1266; and conſequently the part of the power ſpent in producing the effect, is to the greateſt effect producible thereby as 3489 : 1266 :: 10 : 3,62, or as 11 to 4.

The *velocity of the water* ſtriking the wheel has been determined to be equal to 86 circumferences of the wheel per minute, and the *velocity of the wheel* at the *maximum* to be 30; the velocity of the water will therefore be to that of the wheel as 86 to 30, or as 10 to 3,5, or as 20 to 7.

The *load at the maximum* has been ſhown to be equal to 9 lb. 6 oz. and that the wheel ceaſed moveing with 12 lb. in the ſcale: to which if the weight of the ſcale is added, *viz.* 10 ounces *, the proportion will be nearly as 3 to 4 between the load at the *maximum* and *that* by which the wheel is ſtopped.

It is ſomewhat remarkable, that tho' the velocity of the wheel in relation to the water turns out greater than $\frac{1}{3}$ of the velocity of the the water, yet the impulſe of the water in the caſe of a *maximum* is more than double of what is aſſigned by theory; that is, inſtead of $\frac{4}{9}$ of the column, it is nearly equal to the whole column.

It muſt be remembred, therefore, that, in the preſent caſe, the wheel was not placed in an open river, where the natural current, after it has communicated its impulſe to the float, has room on all ſides to eſcape, as the theory ſuppoſes; but in a conduit or

* The reſiſtance of the air in this caſe ceaſes, and the friction is not added, as 12 lb. in the ſcale was ſufficient to ſtop the wheel after it had been in full motion; and therefore ſomewhat more than a counterbalance to the impulſe of the water.

race,

race, to which the float being adapted, the water cannot otherwiſe eſcape than by moving along with the wheel. It is obſervable, that a wheel working in this manner, as ſoon as the water meets the float, receiving a ſudden check, it riſes up againſt the float, like a wave againſt a fixed object; inſomuch that when the ſheet of water is not a quarter of an inch thick before it meets the float, yet this ſheet will act upon the whole ſurface of a float, whoſe height is 3 inches; and conſequently was the float no higher than the thickneſs of the ſheet of water, as the theory alſo ſuppoſes, a great part of the force would have been loſt, by the water's daſhing over the float *.

In further confirmation of what is already delivered, I have adjoined the following table, containing the reſult of 27 ſetts of experiments, made and reduced in the manner above ſpecified. What remains of the theory of underſhot wheels, will naturally follow from a compariſon of the different experiments together.

* Since the above was wrote, I find that Profeſſor Euler, in the Berlin Acts for the year 1748, in a memoire intitled, *Maxims pour aranger le plus avantageuſment les machines deſtinées à elever de l'eau par le moyen de pompes*, page 192. § 9. has the following paſſage; which ſeems to be the more remarkable, as I don't find he has given any demonſtration of the principle therein contained, either from theory or experiment; or has made any uſe thereof in his calculations on this ſubject.——" Cependant dans ce cas puiſque " l'eau eſt reflechie, & qu'elle decoule ſur les aubes vers les cotés, " elle y exerce encore une force particuliere, dont l'effet de l'im- " pulſion ſera augmenté; & experience jointe a la theorie a fait " voir que dans ce cas, la force eſt preſque double: de ſorte " qu'il faut prendre le double de le ſection du fil d'eau pour ce " qui repond dans ce cas a le ſurface des aubes, poûrvu qu'elles " ſoient aſſez larges pour recevoir ce ſupplement de force. Car ſi " les aubes nétoient plus larges que le fil, on trait d'eau on ne " devroit prendre que ne ſimple ſection, tout comme dans le pre- " mier cas, on l'aube toute entire eſt pappee par l'eau."

TABLE I.

N°	Height of the water in the ciftern.	Turns of the wheel unloaded.	Virtual head deduced therefrom.	Turns at the maxim^m.	Load at the equilibrium.		Load at the maximum.		Water expended in a minute.	Power.	Effect.	Ratio of the power and effect.	Ratio of the velocity of the water and wheel.	Ratio of the load at the equilibrium, to the load at the maximum.	Experiments.
	In.		In.		*lb.*	*oz.*	*lb.*	*oz.*							
1	33	88	15,85	30,	13	10	10	9	275,	4358	1411	10:3,24	10:3,4	10:7,75	
2	30	86	15,0	30,	12	10	9	6	264,7	3970	1266	10:3,2	10:3,5	10:7,4	
3	27	82	13,7	28,	11	2	8	6	243,	3329	1044	10:3,15	10:3,4	10:7,5	
4	24	78	12,3	27,7	9	10	7	5	235,	2890	901,4	10:3,12	10:3,55	10:7,53	At the 1ft hole.
5	21	75	11,4	25,9	8	10	6	5	214,	2439	735,7	10:3,02	10:3,45	10:7,32	
6	18	70	9,95	23,5	6	10	5	5	199,	1970	561,8	10:2,85	10:3,36	10:8,02	
7	15	65	8,54	23,4	5	2	4	4	178,5	1524	442,5	10:2,9	10:3,6	10:8,3	
8	12	60	7,29	22,	3	10	3	5	161,	1173	328	10:2,8	10:3,77	10:9,1	
9	9	52	5,47	19,	2	12	2	8	134,	733	213,7	10:2,9	10:3,65	10:9,1	
10	6	42	3,55	16,	1	12	1	10	114,	404,7	117	10:2,82	10:3,8	10:9,3	
11	24	84	14,2	30,75	13	10	10	14	342,	4890	1505	10:3,075	10:3,66	10:7,9	
12	21	81	13,5	29,	11	10	9	6	297,	4009	1223	10:3,01	10:3,62	10:8,05	
13	18	72	10,5	26,	9	10	8	7	285,	2993	975	10:3,25	10:3,6	10:8,75	At the 2d.
14	15	69	9,6	25,	7	10	6	14	277,	2659	774	10:2,92	10:3,62	10:9,	
15	12	63	8,0	25,	5	10	4	14	234,	1872	549	10:2,94	10:3,97	10:8,7	
16	9	56	6,37	23,	4	0	3	13	201,	1280	390	10:3,05	10:4,1	10:9,5	
17	6	46	4,25	21,	2	8	2	4	167,5	712	212	10:2,98	10:4,55	10:9,	
18	15	72	10,5	29,	11	10	9	6	357,	3748	1210	10:3,23	10:4,02	10:8,05	
19	12	66	8,75	26,75	8	10	7	6	330,	2887	878	10:3,05	10:4,05	10:8,1	The 3d.
20	9	58	6,8	24,5	5	8	5	0	255,	1734	541	10:3,01	10:4,22	10:9,1	
21	6	48	4,7	23,5	3	2	3	0	228,	1064	317	10:2,99	10:4,9	10:9,6	
22	12	68	9,3	27,	9	2	8	6	359,	3338	1006	10:3,02	10:3,97	10:9,17	
23	9	58	6,8	26,25	6	2	5	13	332,	2257	686	10:3,04	10:4,52	10:9,5	4th.
24	6	48	4,7	24,5	3	12	3	8	262,	1231	385	10:3,13	10:5,1	10:9,35	
25	9	60	7,29	27,3	6	12	6	6	355,	2588	783	10:3,03	10:4,55	10:9,45	5th.
26	6	50	5,03	24,6	4	6	4	1	307,	1544	450	10:2,92	10:4,9	10:9,3	
27	6	50	5,03	26,	4	15	4	9	360,	1811	534	10:2,95	10:5,2	10:9,25	6th.
1.	2.	3.	4.	5.	6.		7.		8.	9.	10.	11,	12.	13.	

Maxims and Observations deduced from the foregoing Table of Experiments.

Maxim I. *That the virtual or effective head being the same, the effect will be nearly as the quantity of water expended.*

This will appear by comparing the contents of the columns 4, 8, and 10, in the foregoing setts of experiments; as for

Example 1st, taken from N°. 8. *and* 25, viz.

N°.	Virtual Head.	Water expended.	Effect.
8	7,29	161	328
25	7,29	355	785

Now the heads being equal, if the effects are proportioned to the water expended, we shall have by maxim 1st, 161 : 355 :: 328 : 723; but 723 falls short of 785, as it turns out in experiment according to N°. 25, by 62; the effect therefore of N°. 25, compared with N°. 8, is greater than according to the present maxim in the ratio of 14 to 13.

The foregoing example, with four similar ones, are seen at one view in the following Table.

Examples	N°Tab.I.	Virtual Head.	Expence of Water.	Effect.	Comparison.	Variation	Proportional Variation.
		Inch.	*lb.*				
1st	8 25	7,29 7,29	161 355	328 785	161 : 355 :: 328 : 723	62+	14 : 13
2d	13 18	10,5 10,5	285 357	975 1210	285 : 357 :: 975 : 1221	11—	121:122
3d	22 23	6,8 6,8	255 332	541 686	255 : 332 :: 541 : 704	18—	38 : 39
4th	21 24.	4,7 4,7	228 262	317 385	228 : 262 :: 317 : 364	21+	18 : 17
5th	26 27	5,03 5,03	307 360	450 534	307 : 360 :: 450 : 531	3 +	178:177

Hence

Hence therefore, in comparing different experiments, as ſome fall ſhort, and others exceed the maximum, and all agree therewith, as near as can be expected, in an affair where ſo many different circumſtances are concerned; we may, according to the laws of reaſoning by induction, conclude the maxim true; *viz.* that the effects are nearly as the quantity of water expended.

Maxim II. *That the expence of water being the ſame, the effect will be nearly as the height of the virtual or effective head.*

This alſo will appear by comparing the contents of columns 4, 8, and 10, in any of the ſetts of experiments.

Example 1ſt, of N°. 2. *and* N°. 24. *viz.*

N°.	Virt. Head.	Expence.	Effect.
2	15	264,7	1266
24	4,7	262	385

Now as the expences are not quite equal, we muſt proportion one of the effects accordingly: thus

by maxim 1ſt, 262 : 264,7 :: 385 : 389
and by max. 2d, 15 : 4,7 :: 1266 : 397

Difference —— 8

The effect therefore of N°. 24. compared with N°. 2. is leſs than according to the preſent maxim in the ratio of 49 : 50.

The foregoing, and two other ſimilar examples, are compriſed in the following Table.

Examples

Examples.	N°. Tab. I.	Virtual Head.	Expence of Water.	Effect.	Comparison.	Variation.	Proportional Variation.
1st	2 24	15 4,7	264,7 262	1266 385	Max. 1st, 262 : 264,7 :: 385 : 319 Max. 2d, 15 : 4,7 :: 1266 : 397	8 —	49 : 50
2d	1 10	15,85 3,55	275 114	1411 117	Max. 1st, 114 : 275 :: 117 : 282 Max. 2d. 15,85 : 3,55 :: 1411 : 316	34—	8 : 9
3d	11 17	14,2 4,25	342 1675	1505 212	Max. 1st, 167,5 : 342 :: 212 : 433 Max. 2d, 14,2 : 4,25 :: 1505 : 450	17—	25 : 26

Maxim III. *That the quantity of water expended being the ſame, the effect is nearly as the ſquare of its velocity.*

This will appear by comparing the contents of columns 3, 8, and 10, in any of the ſetts of experiments; as for

Example 1ſt of N°. 2. *with* N°. 24. *viz.*

N°.	Turns in a min.	Expence.	Effect.
2	86	264,7	1266
24	48	262	385

The velocity being as the number of turns, we ſhall have,

by max. 1ſt, 262 : 264,7 :: 385 : 389

and by max. 3d, $\left\{\begin{matrix} 86^2 : 48^2 \\ 7396 : 2304 \end{matrix}\right\}$ :: 1266 : 394

Difference - - - 5

The effect therefore of N°. 24. compared with N°. 2. is leſs than by the preſent maxim in the ratio of 78 : 79.

The foregoing, and three other ſimilar examples, are compriſed in the following Table.

Examples.

Examples.	No. Tab. I.	Turns in a minute.	Expence of Water.	Effect.	Comparison.	Variation.	Proportional Variation.
1st	2	86	264,7	1266	Max. 1st, 262 : 264,7 :: 385 : 389	5 —	78 : 79
	24	48	262	385	Max. 3d, $\left\{ \begin{array}{c} 86^2 : 48^2 \\ 7396 : 2304 \end{array} \right\}$ :: 1266 : 394		
2d	1	88	275	1411	Max. 1st, 114 : 275 :: 117 : 282	39 —	7 : 8
	10	42	114	117	Max. 3d, $\left\{ \begin{array}{c} 88^2 : 42^2 \\ 7744 : 1764 \end{array} \right\}$ :: 1411 : 321		
3d	11	84	342	1505	Max. 1st, 167,5 : 342 :: 212 : 433	18 —	24 : 25
	17	46	167,5	212	Max. 3d, $\left\{ \begin{array}{c} 84^2 : 46^2 \\ 7056 : 2116 \end{array} \right\}$ :: 1505 : 451		
4th	18	72	357	1210	Max. 1st, 228 : 357 :: 317 : 496	42 —	12 : 13
	21	48	228	317	Max. 3d, $\left\{ \begin{array}{c} 72^2 : 48^2 \\ 5184 : 2301 \end{array} \right\}$ :: 1210 : 538		

Maxim

Maxim 4th. *The aperture being the ſame, the effect will be nearly as the cube of the velocity of the water.*

This alſo will appear by comparing the contents of columns 3, 8, and 10; as for

Example 1ſt, of N°. 1. *and* N°. 10, *viz.*

N°.	Turns.	Expence.	Effect.
1	88	275	1411
10	42	114	117

Lemma. It muſt here be obſerved, that if water paſſes out of an aperture, in the ſame ſection, but with different velocities; the expence will be proportional to the velocity; and therefore converſely, if the expence is not proportional to the velocity, the ſection of the water is not the ſame.

Now comparing the water diſcharged with the turns of N°. 1. and 10, we ſhall have 88 : 42 :: 275 : 131,2; but the water diſcharged by N°. 10. is only 114 lb. therefore, tho' the ſluice was drawn to the ſame height in N°. 10. as in N°. 1. yet the ſection of the water paſſing out, was leſs in N°. 10. than N°. 1. in the proportion of 114 to 131,2; conſequently had the effective aperture or ſection of the water been the ſame in N°. 10. as in N°. 1. ſo that 131,2 lb. of water had been diſcharged inſtead of 114, the effect would have been increaſed in the ſame proportion; that is,

by the *Lemma*, 88 : 42 :: 275 : 131,2

by maxim 1ſt, 114 : 131,2 :: 117 : 134,5

and by max. 4th, $\left\{\begin{matrix} 83^3 : 42^3 \\ 681472 : 74088 \end{matrix}\right\}$ :: 1411 : 153,5

Difference —— 19

The

The effect therefore of N°. 10. compared with N°. 1. is less than it ought to be by the present maxim in the ratio of 7 : 8.

The foregoing, and three other similar examples, are contained in the following Table.

Examples.	N°. Tab. I.	Turns in a minute.	Expence of water.	Effect.	Comparison.	Variation.	Proportional Variation.
1st	1 10	88 42	275 114	1411 117	Lemma. 88 : 42 :: 275 : 131,2 Max. 1. 114 : 131,2 :: 117 : 134,5 Max. 4. $88^3 : 42^3$ :: 1411 : 153,5	19—	7 : 8
2d	11 17	84 46	342 167,5	1505 212	Lemma. 84 : 46 :: 342 : 187,3 Max. 1. 167,5 : 187,3 :: 212 : 237 Max. 4. $84^3 : 46^3$ :: 1505 : 247	10—	23 : 24
3d	18 21	72 48	357 228	1210 317	Lemma. 72 : 48 :: 357 : 238 Max. 1. 228 : 238 :: 317 : 331 Max. 4. $72^3 : 48^3$ :: 1210 : 355	24—	14 : 15
4th	22 24	68 48	359 262	1006 385	Lemma. 68 : 48 :: 359 : 253,4 Max. 1. 262 : 253,4 :: 385 : 372 Max. 4. $68^3 : 48^3$ :: 1006 : 354	18+	20 : 19

OBSERVATIONS.

Obſerv. 1*ſt.* On comparing column 2d and 4th, Tab. I. it is evident, that the *virtual head* bears no certain proportion to the *head of water*; but that when the aperture is greater, or the velocity of the water iſſuing therefrom leſs, they approach nearer to a coincidence: and conſequently in the large openings of mills and ſluices, where great quantities of water are diſcharged from moderate heads, the head of water, and virtual head determined from the velocity, will nearly agree, as experience confirms.

Obſerv. 2*d.* Upon comparing the ſeveral proportions between the *power* and *effect* in column 11th, the moſt general is that of 10 to 3; the extremes 10 to 3,2 and 10 to 2,8; but as it is obſervable, that where the quantity of water, or the velocity thereof; that is, where the power is greateſt, the 2d term of the ratio is greateſt alſo: we may therefore well allow the proportion ſubſiſting in large works, as 3 to 1.

Obſerv. 3*d.* The proportions of *velocities* beween the *water* and *wheel* in column 12, are contained in the limits of 3 to 1 and 2 to 1; but as the greater velocities approach the limit of 3 to 1, and the greater quantity of water approach to that of 2 to 1, the beſt general proportion will be that of 5 to 2.

Obſerv. 4*th.* On comparing the numbers in column 13, it appears, that there is no certain ratio between the *load* that the wheel will carry at its *maximum*, and what will totally ſtop it; but that they are contained within the limits of 20 to 19, and

of 20 to 15; but as the effect approaches nearest to the ratio of 20 to 15, or of 4 to 3, when the power is greatest, whether by increase of velocity, or quantity of water, this seems to be the most applicable to large works: but as the load that a wheel ought to have, in order to work to the best advantage, can be assigned, by knowing the effect it ought to produce, and the velocity it ought to have in producing it; the exact knowlege of the greatest load it will bear, is of the less consequence in practice.

It is to be noted, that in all the examples under the three last of the four preceding maxims, the effect of the lesser power falls short of its due proportion to the greater, when compared by its maxim; except the last example of maxim 4th: and hence, if the experiments are taken strictly, we must infer, that the effects increase and diminish in an higher ratio than those maxims suppose: but as the deviation is not very considerable, the greatest being about 1-8th of the quantity in question; and as it is not easy to make experiments of so compounded a nature with absolute precision; we may rather suppose, that the lesser power is attended with some friction, or works under some disadvantage, which has not been duly accounted for, and therefore we may conclude, that these maxims will hold very nearly, when applied to works in large.

After the experiments above mentioned were tried, the wheel, which had originally 24 floats, was reduced to twelve; which caused a diminution in the effect, on account of a greater quantity of water escaping between the floats and the floor; but a circular

cular ſweep being adapted thereto, of ſuch a length, that one float entered the curve before the preceding one quitted it, the effect came ſo near to the former, as not to give hopes of advancing it by increaſing the number of floats beyond 24 in this particular wheel.

PART II.

Concerning OVERSHOT WHEELS.

Read May 24, 1759.

IN the former part of this eſſay, we have conſidered the impulſe of a confined ſtream, acting on *Underſhot Wheels.* We now proceed to examine the power and application of water, when acting by its *gravity* on *Overſhot Wheels.*

In reaſoning without experiment, one might be led to imagine, that however different the mode of application is; yet that whenever the ſame quantity of water deſcends thro' the ſame perpendicular ſpace, that the natural effective power would be equal; ſuppoſing the machinery free from friction, equally calculated to receive the full effect of the power, and to make the moſt of it: for if we ſuppoſe the height of a column of water to be 30 inches, and reſting upon a baſe or aperture of one inch ſquare; every cubic inch of water that departs therefrom will acquire the ſame velocity or *momentum*, from the uniform preſſure of 30 cubic inches above it, that one cubic inch let fall from the top will acquire in falling down to the level of the aperture; *viz.* ſuch a velocity as in a contrary direction would carry it to the

the level from whence it fell; * one would therefore ſuppoſe, that a cubic inch of water, let fall thro' a ſpace of 30 inches, and there impinging upon another body, would be capable of producing an equal effect by colliſion, as if the ſame cubic inch had deſcended thro' the ſame ſpace with a ſlower motion, and produced its effects gradually: for in both caſes gravity acts upon an equal quantity of matter, thro' an equal ſpace †; and conſequently, that whatever was the ratio between the power and effect in underſhot wheels, the ſame would obtain in overſhot, and indeed in all others: yet, however concluſive this reaſoning may ſeem, it will appear, in the courſe of the following deductions, that the effect of the gravity of deſcending bodies is very different from the effect of the ſtroke of ſuch as are *non-elaſtic*, tho' generated by an equal mechanical power.

The alterations in the machinery already deſcribed, to accommodate the ſame for experiments on overſhot wheels, were principally as follows.

PLATE V. *Fig.* 2. The ſluice I *b* being ſhut down, the rod H I was unſcrewed and taken off.

The underſhot water-wheel was taken off the axis, and inſtead thereof an overſhot wheel of the ſame

* This is a conſequence of the riſing of jetts to the height of their reſervoirs nearly.

† Gravity, it is true, acts a longer ſpace of time upon the body that deſcends ſlow than upon that which falls quick; but this cannot occaſion the difference in the effect: for an elaſtic body falling thro' the ſame ſpace in the ſame time, will, by colliſion upon another elaſtic body, rebound nearly to the height from which it fell; or, by communicating its motion, cauſe an equal one to aſcend to the ſame height.

diameter

diameter was put into its place. *Note*, This wheel was two inches in the ſhroud or depth of the bucket; the number of the buckets was 36.

The ſtandards S and T, *Fig.* 1. were raiſed half an inch, ſo that the bottom of the wheel might be clear of ſtagnant water.

A trunk, for bringing the water upon the wheel, was fixed according to the dotted lines *f g*, *Fig.* 2. The aperture was adjuſted by a ſhuttle *h i*, which alſo cloſed up the outer end of the trunk, when the water was to be ſtopped.

Fig. 3. The ratchet *o o*, not being of one piece of metal with the ferrule *e e*, *i i* (tho' ſo deſcribed before, to prevent unneceſſary diſtinctions), was with its catch turned the contrary ſide; conſequently the moveable barrel would do its office equally, notwithſtanding the water-wheel, when at work, moved the contrary way.

Specimen of a Sett of Experiments.

Head 6 inches.

14 ½ ſtrokes of the pump in a minute, 12 ditto = 80 lb. *

Weight of the ſcale (being wet) 10 ½ oz.

Counterweight for 20 turns, beſides the ſcale, 3 oz.

No.	Weight in the Scale.	Turns.	Product.	Obſervations.
1	0 *lb.*	60	——	Threw moſt part of the water out of the wheel.
2	1	56	——	
3	2	52	——	
4	3	49	147	Received the water more quietly.
5	4	47	188	
6	5	45	225	
7	6	42 ½	255	
8	7	41	287	
9	8	38 ½	308	
10	9	36 ½	328 ½	
11	10	35 ½	355	
12	11	32 ¾	360 ½	
13	12	31 ¼	375	
14	13	28 ½	370 ½	
15	14	27 ½	385	
16	15	26	390	
17	16	24 ½	392	
18	17	22 ¾	386 ¾	
19	18	21 ¾	391 ½	
20	19	20 ¾	394 ¼	Maximum.
21	20	19 ¾	395	
22	21	18 ¼	388 ¼	
23	22	18	396	Work'd irregular.
24	23	Overſet by its load.		

* The ſmall difference, in the value of 12 ſtrokes of the pump, from the former experiments, was owing to a ſmall difference in the length of the ſtroke, occaſioned by the warping of the wood.

Reduction

Reduction of the preceding Specimen.

In theſe experiments the head being 6 inches, and the height of the wheel 24 inches, the whole deſcent will be 30 inches: the expence of water was $14\frac{1}{2}$ ſtrokes of the pump in a minute, whereof 12 contained 80 lb.; therefore the water expended in a minute was $96\frac{2}{3}$ lb. which, multiplied by 30 inches, gives the *power* = 2900.

If we take the 20th experiment for the *maximum*, we ſhall have $20\frac{3}{4}$ turns in a minute, each of which raiſed the weight $4\frac{1}{2}$ inches, that is, 93,37 inches in a minute. The weight *in* the ſcale was 19 lb, the weight *of* the ſcale $10\frac{1}{2}$ oz.; the counter-weight 3 oz. in the ſcale, which, with the weight of the ſcale $10\frac{1}{2}$ oz. makes in the whole $20\frac{1}{2}$ lb. which is the whole reſiſtance or load: this, multiplied by 93,37 inches, makes 1914 for the effect.

The *ratio* therefore of the *power* and *effect* will be as 2900 : 1914, or as 10 : 6,6, or as 3 : 2 nearly.

But if we compute the power from the height of the wheel only, we ſhall have $96\frac{2}{3}$ lb. multiplied by 24 inches = 2320 for the *power*, and this will be to the *effect* as 2320 : 1914, or as 10 : 82, or as 5 : 4 nearly.

The reduction of this ſpecimen is ſet down in N°. 9. of the following Table; and the reſt were deducted from a ſimilar ſett of experiments, reduced in the ſame manner.

TABLE II. *containing the Result of Sixteen Setts of Experiments on Overshot Wheels.*

N°.	Whole descent.	Water expended in a minute.	Turns at the maximum in a min.	Weight raised at the maxim^{m}.	Power of the whole descent.	Power of the wheel.	Effect.	Ratio of the whole power and effect.	Ratio of power of the wheel and effect.	Mean ratio.
	Inch.	*lb.*		*lb.*						
1	27	30	19	6½	810	720	556	10 : 6,9	10 : 7,7	Medium 10 : 8,1
2	27	56⅔	16¼	14½	1530	1360	1060	10 : 6,9	10 : 7,8	
3	27	56⅔	20¾	12½	1530	1360	1167	10 : 7,6	0 : 8,4	
4	27	63⅓	20½	13½	1710	1524	1245	10 : 7,3	10 : 8,2	
5	27	76⅔	21½	15½	2070	1840	1500	10 : 7,3	10 : 8,2	
6	28½	73⅓	18¾	17½	2090	1764	1476	10 : 7,	10 : 8,4	10 : 8,2
7	28½	96⅔	20¼	20½	2755	2320	1868	10 : 6,8	10 : 8,	
8	30	90	20	19½	2700	2160	1755	10 : 6,5	10 : 8,1	10 : 8,2
9	30	96⅔	20¾	20½	2900	2320	1914	10 : 6,6	10 : 8,2	
10	30	113⅓	21	23½	3400	2720	2221	10 : 6,5	10 : 8,2	
11	33	56⅔	20¼	13½	1870	1360	1230	10 : 6,6	10 : 9,	10 : 8,5
12	33	106⅔	22¼	21½	3520	2560	2153	10 : 6,1	10 : 8,4	
13	33	146⅔	23	27½	4840	3520	2846	10 : 5,9	10 : 8,1	
14	35	65	19¾	16½	2275	1560	1466	10 : 6,5	10 : 9,4	10 : 8,5
15	35	120	21½	25½	4200	2880	2467	10 : 5,9	10 : 8,6	
16	35	163½	25	26½	5728	3924	2981	10 : 5,2	10 : 7,6	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11

Observations and Deductions from the foregoing Experiments.

I. *Concerning the Ratio between the Power and Effect of Overshot Wheels.*

The effective power of the water must be reckoned upon the whole descent; because it must be

raised

raifed that height, in order to be in a condition of producing the fame effect a fecond time.

The ratio's between the *powers* fo eftimated, and the *effects* at the *maximum* deduced from the feveral fetts of experiments, are exhibited at one view in column 9. of Table II.; and from hence it appears, that thofe ratio's differ from that of 10 to 7,6 to that of 10 : 5,2, that is, nearly from 4 : 3 to 4 : 2. In thofe experiments where the heads of water and quantities expended are leaft, the proportion is nearly as 4 : 3; but where the heads and quantities are greateft, it approaches nearer to that of 4 : 2; and by a medium of the whole, the ratio is that of 3 : 2 nearly. We have feen before, in our obfervations upon the effects of underfhot wheels, that the general ratio of the power to the effect, when greateft, was 3 : 1; *the effect therefore of overfhot wheels, under the fame circumftances of quantity and fall, is at a medium double to that of the underfhot:* and, as a confequence thereof, *that nonelaftic bodies, when acting by their impulfe or collifion, communicate only a part of their original power*; the other part being fpent in changing their figure in confequence of the ftroke.

The powers of water computed from the height of the wheel only, compared with the effects, as in column 10. appear to obferve a more conftant ratio: for if we take the medium of each clafs, which is fet down in column 11, we fhall find the extremes to differ no more than from the ratio of 10 : 8,1 to that of 10 : 8,5; and as the fecond term of the ratio gradually increafes from 8,1 to 8,5, by an increafe of head from 3 inches to 11, the excefs of 8,5 above

8,1

8,1 is to be imputed to the ſuperior impulſe of the water at the head of 11 inches above that of 3 inches: ſo that if we reduce 8,1 to 8, on account of the impulſe of the 3 inch head, *we ſhall have the ratio of the power, computed upon the height of the wheel only, to the effect at a maximum as* 10 : 8, *or as* 5 : 4 *nearly:* and from the equality of the ratio between power and effect, ſubſiſting where the conſtructions are ſimilar, we muſt infer, *that the effects,* as well as the powers, *are as the quantities of water and perpendicular heights multiplied together reſpectively.*

II. *Concerning the moſt proper Height of the Wheel in proportion to the whole Deſcent.*

We have already ſeen, from the preceding obſervation, that the effect of the ſame quantity of water, deſcending thro' the ſame perpendicular ſpace, is double, when acting by its gravity upon an overſhot wheel, to what the ſame produces when acting by its impulſe upon an underſhot. It alſo appears, that by increaſing the head from 3 inches to 11, that is, the whole deſcent, from 27 inches to 35, or in the ratio of 7 to 9 nearly, the effect is advanced no more than in the ratio of 8,1 to 8,4, that is, as 7 : 7,26; and conſequently the increaſe of effect as not 1-7th of the increaſe of perpendicular height. Hence it follows, *that the higher the wheel is in proportion to the whole deſcent, the greater will be the effect;* becauſe it depends leſs upon the impulſe of the head, and more upon the gravity of the water in the buckets: and if we conſider how obliquely the water iſſuing from the head muſt ſtrike the buckets, we ſhall not be at a loſs to account for the little ad-

vantage that ariſes from the impulſe thereof; and ſhall immediately ſee of how little conſequence this impulſe is to the effect of an overſhot wheel. However, as every thing has its limits, ſo has this: for thus much is deſirable, *that the water ſhould have ſomewhat greater velocity, than the circumference of the wheel, in coming thereon*; otherwiſe the wheel will not only be retarded, by the buckets ſtriking the water, but thereby daſhing a part of it over, ſo much of the power is loſt.

The velocity that the circumference of the wheel ought to have, being known by the following deductions, the head requiſite to give the water its proper velocity is eaſily computed from the common rules of hydroſtatics; and will be found much leſs than what is generally practiſed.

III. *Concerning the Velocity of the Circumference of the Wheel, in order to produce the greateſt Effect.*

If a body is let fall freely from the ſurface of the head to the bottom of the deſcent, it will take a certain time in falling; and in this caſe the whole action of gravity is ſpent in giving the body a certain velocity: but if this body in falling is made to act upon ſome other body, ſo as to produce a mechanical effect, the falling body will be retarded; becauſe a part of the action of gravity is then ſpent in producing the effect, and the remainder only giving motion to the falling body: and therefore *the ſlower a body deſcends, the greater will be the portion of the action of gravity applicable to the producing a mechanical effect*; and in conſequence the greater that effect may be.

If a ſtream of water falls into the bucket of an overſhot wheel, it is there retained till the wheel by moving round diſcharges it: of conſequence the ſlower the wheel moves, the more water each bucket will receive: ſo that what is loſt in ſpeed, is gained by the preſſure of a greater quantity of water acting in the buckets at once: and, if conſidered only in this light, the mechanical power of an overſhot wheel to produce effects will be equal, whether it moves quick or ſlow: but if we attend to what has been juſt now obſerved of the falling body, it will appear that ſo much of the action of gravity, as is employed in giving the wheel and water therein a greater velocity, muſt be ſubtracted from its preſſure upon the buckets; ſo that, tho' the product made by multiplying the number of cubic inches of water acting in the wheel at once by its velocity will be the ſame in all caſes; yet, as each cubic inch, when the velocity is *greater* does not preſs ſo much upon the bucket as when it is *leſs*, the power of the water to produce effects will be greater in the leſs velocity than in the greater: and hence we are led to this general rule, *that*, cæteris paribus, *the leſs the velocity of the wheel, the greater will be the effect thereof.* A confirmation of this doctrine, together with the limits it is ſubject to in practice, may be deduced from the foregoing ſpecimen of a ſett of experiments.

From theſe experiments it appears, that when the wheel made about 20 turns in a minute, the effect was, near upon, the greateſt. When it made 30 turns, the effect was diminiſhed about $\frac{1}{20}$ part; but that when it made 40, it was dimimiſhed about $\frac{1}{4}$; when it made leſs than 18 $\frac{1}{4}$, its motion was irregular; and when

when it was loaded ſo as not to admit its making 18 turns, the wheel was overpowered by its load.

It is an advantage in practice, that the velocity of the wheel ſhould not be diminiſhed further than what will procure ſome ſolid advantage in point of power; becauſe, *cæteris paribus*, as the motion is ſlower, the buckets muſt be made larger; and the wheel being more loaded with water, the ſtreſs upon every part of the work will be increaſed in proportion: *The beſt velocity for practice therefore will be ſuch, as when the wheel here uſed made about* 30 *turns in a minute*; *that is, when the velocity of the circumference is a little more than* 3 *feet in a ſecond.*

Experience confirms, that this velocity of 3 feet in a ſecond is applicable to the higheſt overſhot wheels, as well as the loweſt; and all other parts of the work being properly adapted thereto, will produce very nearly the greateſt effect poſſible: however this alſo is certain from experience, that *high wheels may deviate further from this rule, before they will loſe their power, by a given aliquot part of the whole, than low ones can be admitted to do*; for a wheel of 24 feet high may move at the rate of ſix feet per ſecond without loſing any conſiderable part of its power*; and, on the other hand, I have ſeen a wheel of 33 feet high, that has moved very ſteadily and well with a velocity but little exceeding 2 feet.

* The 24 feet wheel going at 6 feet in a ſecond ſeems owing to the ſmall proportion that the head (requiſite to give the water the proper velocity of the wheel) bears to the whole height.

IV. *Concerning the Load for an Overſhot Wheel, in order that it may produce a Maximum.*

The maximum load for an overſhot wheel, is that which reduces the circumferences of the wheel to its proper velocity; and this will be known, by dividing the effect it ought to produce in a given time by the ſpace intended to be deſcribed by the circumference of the wheel in the ſame time: the quotient will be the reſiſtance overcome at the circumference of the wheel; and is equal to the load required, the friction and reſiſtance of the machinery included.

V. *Concerning the greateſt poſſible Velocity of an Overſhot Wheel.*

The greateſt velocity that the circumference of an overſhot wheel is capable of, depends jointly upon the diameter or *height* of the wheel, and the velocity of falling bodies; for it is plain that the velocity of the circumference can never be greater, than to deſcribe a ſemi-circumference, while a body let fall from the top of the wheel will deſcend thro' its diameter; nor indeed quite ſo great, as a body deſcending thro' the ſame perpendicular ſpace cannot perform the ſame in ſo ſmall a time when paſſing thro' a ſemi-circle, as would be done in a perpendicular line. Thus, if a wheel is 16 feet 1 inch high, a body will fall thro' the diameter in one ſecond: this wheel therefore can never arrive at a velocity equal to the making one turn in two ſeconds; but, in reality, an overſhot wheel can never come near this velocity; for when it acquires a certain ſpeed, the

the greateſt part of the water is prevented from entering the buckets; and the reſt, at a certain point of its deſcent, is thrown out again by the centrifugal force. This appears to have been the caſe in the three firſt experiments of the foregoing ſpecimen; but as the velocity, when this begins to happen, depends upon the form of the buckets, as well as other circumſtances, *the utmoſt velocity of overſhot wheels is not to be determined generally:* and, indeed, it is the leſs neceſſary in practice, as it is in this circumſtance incapable of producing any *mechanical effect*, for reaſons already given.

VI. *Concerning the greateſt Load that an Overſhot Wheel can overcome.*

The greateſt load an overſhot wheel will overcome, conſidered abſtractedly, is unlimited or infinite: for as the buckets may be of any given capacity, the more the wheel is loaded, the ſlower it turns; but the ſlower it turns, the more will the buckets be filled with water; and conſequently tho' the diameter of the wheel, and quantity of water expended, are both limited, yet no reſiſtance can be aſſigned, which it is not able to overcome: but in practice we always meet with ſomething that prevents our getting into infiniteſimals; for when we really go to work to build a wheel, the buckets muſt neceſſarily be of ſome given capacity; and conſequently *ſuch a reſiſtance will ſtop the wheel, as is equal to the effort of all the buckets in one ſemi-circumference filled with water.*

The ſtructure of the buckets being given, the quantity of this effort may be aſſigned; but is not of much conſequence to the practice, as in this caſe

alſo

alſo the wheel loſes its power; for tho' here is the exertion of gravity upon a given quantity of water, yet being prevented by a counterbalance from moving, is capable of producing no *mechanical effect*, according to our definition. But, in reality, an overſhot wheel generally ceaſes to be uſeful before it is loaded to that pitch; for *when it meets with ſuch a reſiſtance as to diminiſh its velocity to a certain degree, its motion becomes irregular; yet this never happens till the velocity of the circumference is leſs than 2 feet per ſecond, where the reſiſtance is equable*, as appears not only from the preceding ſpecimen, but from experiments on larger wheels.

SCHOLIUM.

Having now examined the different effects of the power of water, when acting by its *impulſe*, and by its *weight*, under the titles of *underſhot* and *overſhot* wheels; we might naturally proceed to examine the effects when the impulſe and weight are combined, as in the ſeveral kinds of *breaſt-wheels*, *&c.* but, what has been already delivered being carefully attended to, the application of the ſame principles in theſe mixt caſes will be eaſy, and reduce what I have to ſay on this head into a narrow compaſs: for all kinds of wheels where the water cannot deſcend thro' a given ſpace, unleſs the wheel moves therewith, are to be conſidered of the nature of an overſhot wheel, according to the perpendicular height that the water deſcends from; and all thoſe that receive the impulſe or ſhock of the water, whether in an horizontal, perpendicular, or oblique direction, are to be conſidered as underſhots. And therefore a wheel, which the

water ſtrikes at a certain point below the ſurface of the head, and after that deſcends in the arch of a circle, preſſing by its gravity upon the wheel; *the effect of ſuch a wheel will be equal to the effect of an underſhot, whoſe head is equal to the difference of level between the ſurface of the water in the reſervoir and the point where it ſtrikes the wheel, added to that of an overſhot, whoſe height is equal to the difference of level, between the point where it ſtrikes the wheel and the level of the tail-water.* It is here ſuppoſed, that the wheel receives the ſhock of the water at right angles to its radii; and that the velocity of its circumference is properly adapted to receive the utmoſt advantage of both theſe powers; otherwiſe a reduction muſt be made on that account.

Many obvious and conſiderable improvements upon the common practice naturally offer themſelves, from a due conſideration of the principles here eſtabliſhed, as well as many popular errors ſhow themſelves in view: but as my preſent purpoſe extends no farther than the laying down ſuch general rules as will be found to anſwer in practice, I leave the particular application to the intelligent artiſt, and to the curious in theſe matters.

PART III.

On the Conſtruction and Effects of WINDMILL-SAILS.

Read 31 May & 14 June, 1759.

IN trying experiments on windmill-ſails, the wind itſelf is too uncertain to anſwer the purpoſe: we muſt therefore have recourſe to an artificial wind.

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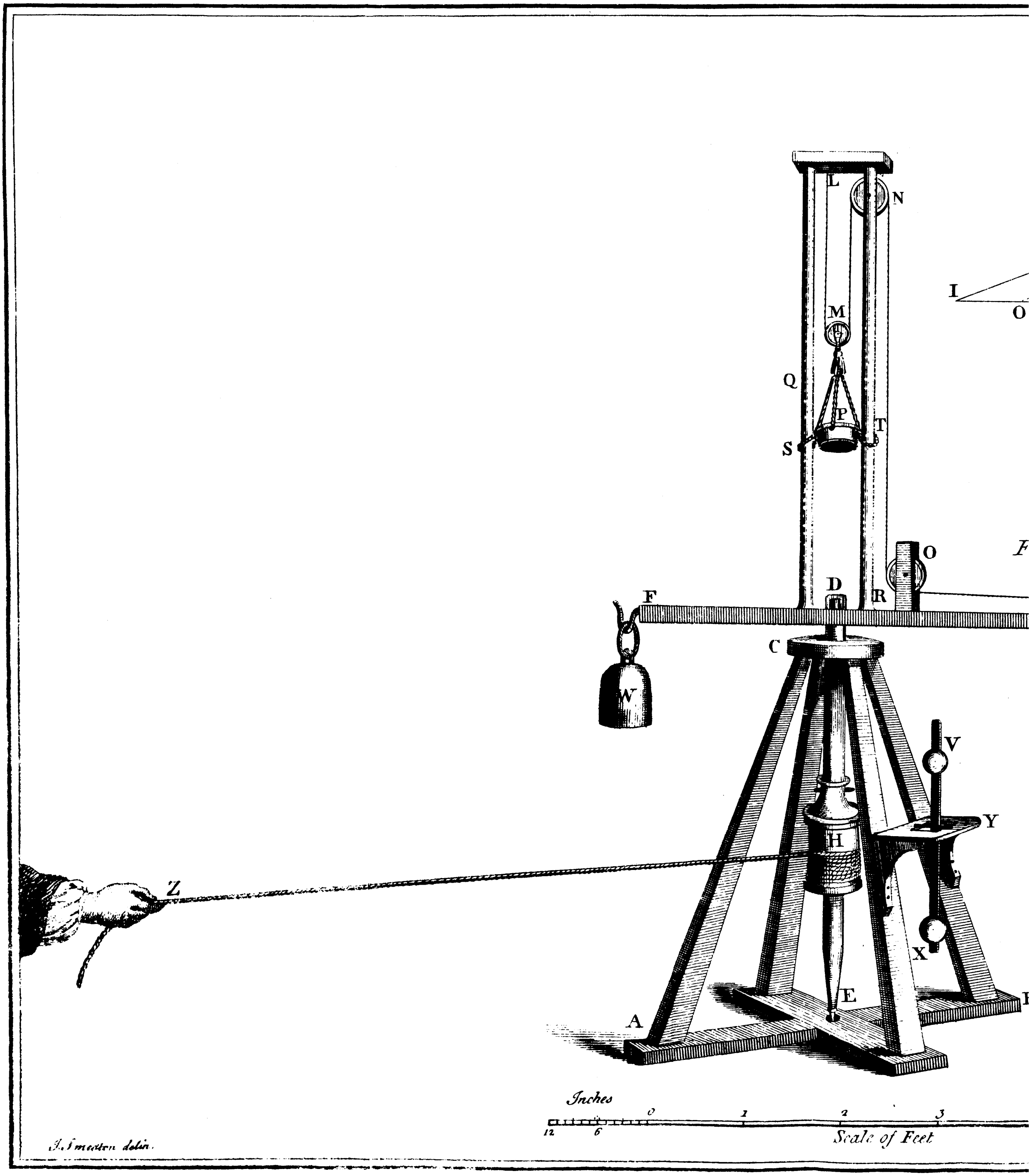

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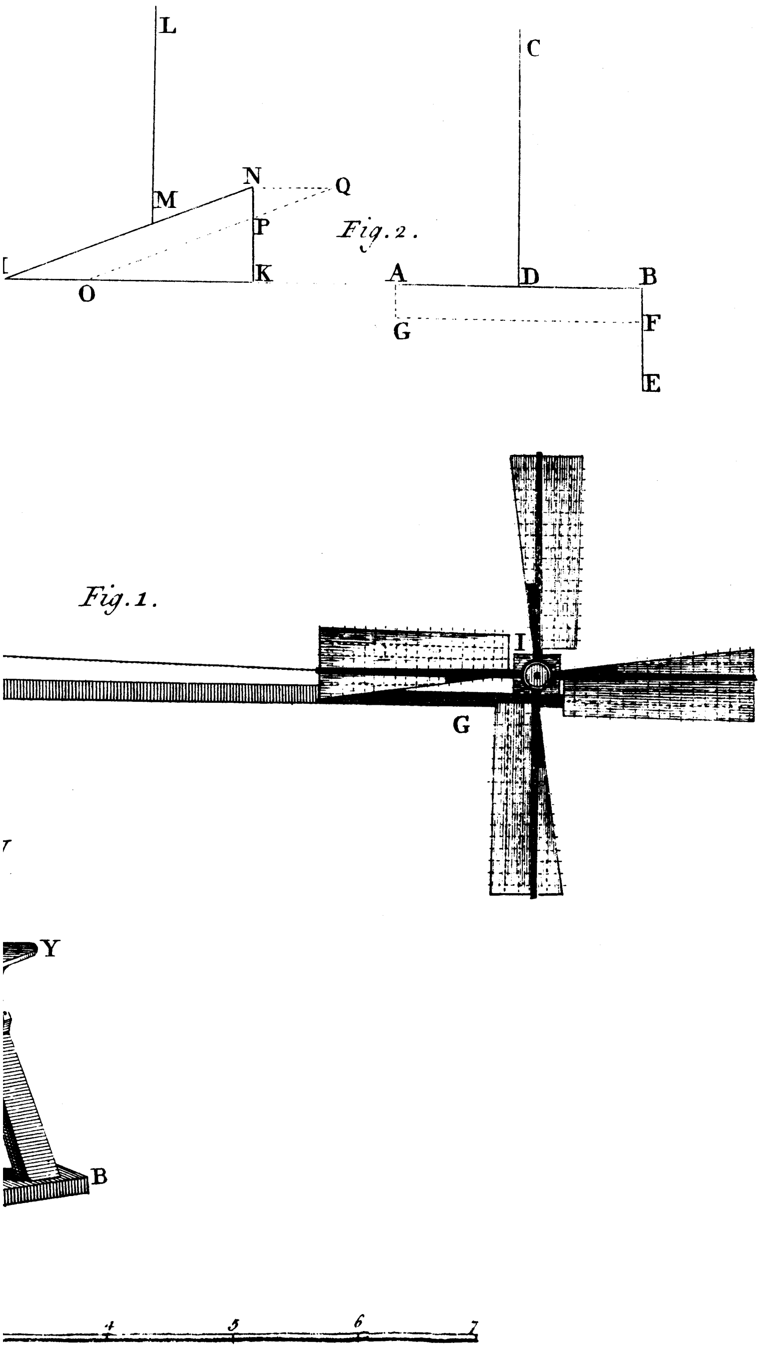

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This may be done two ways; either by cauſing the air to move againſt the machine, or the machine to move againſt the air. To cauſe the air to move againſt the machine, in a ſufficient volumn, with ſteadineſs and the requiſite velocity, is not eaſily put in practice: To carry the machine forward in a right line againſt the air, would require a larger room than I could conveniently meet with. What I found moſt practicable, therefore, was, to carry the axis, whereon the ſails were to be fixed, progreſſively round in the circumference of a large circle. Upon this idea * a machine was conſtructed, as follows.

PLATE VI. *Fig.* 1.

ABC is a pyramidical frame for ſupporting the moving parts.

D E is an upright axis, whereon is framed

F G, an arm for carrying the ſails at a proper diſtance from the center of the upright axis.

* Some years ago Mr. Rouſe, an ingenious gentleman of Harborough in Leiceſterſhire, ſet about trying experiments on the velocity of the wind, and force thereof upon plain ſurfaces and windmill-ſails: and much about the ſame time Mr. Ellicott contrived a machine for the uſe of the late celebrated Mr. B. Robins, for trying the reſiſtance of plain ſurfaces moving thro' the air. The machines of both theſe gentlemen were much alike, tho' at that time totally unacquainted with each other's inquiries. But it often happens, that when two perſons think juſtly upon the ſame ſubject, their experiments are alike. This machine was alſo built upon the ſame idea as the foregoing; but differed in having the hand for the firſt mover, with a pendulum for its regulator, inſtead of a weight, as in the former; which was certainly beſt for the purpoſes of meaſuring the impulſe of the wind, or reſiſtance of plains: but the latter is more applicable to experiments on windmill-ſails; becauſe every change of poſition of the ſame ſails wlll occaſion their meeting the air with a different velocity, tho' urged by the ſame weight.

H is a barrel upon the upright axis, whereon is wound a cord; which, being drawn by the hand, gives a circular motion to the axis, and to the arm F G; and thereby carries the axis of the ſails in the circumference of a circle, whoſe radius is D I, cauſing thereby the ſails to ſtrike the air, and turn round upon their own axis.

At L is fixed the end of a ſmall line, which paſſing through the pullies M N O, terminates upon a ſmall cylinder or barrel upon the axis of the ſails, and, by winding thereon, raiſes

P the ſcale, wherein the weights are placed for trying the power of the ſails. This ſcale, moving up and down in the direction of the upright axis, receives no diſturbance from the circular motion.

Q R two parallel pillars ſtanding upon the arm F G, for the purpoſe of ſupporting and keeping ſteady the ſcale P; which is kept from ſwinging by means of

S T two ſmall chains, which hang looſely round the two pillars.

W is a weight, for bringing the center of gravity of the moveable part of the machine into the center of motion of the axis D E.

V X is a pendulum, compoſed of two balls of lead, which are moveable upon a wooden rod, and thereby can be ſo adjuſted, as to vibrate in any time required. This pendulum hangs upon a cylindrical wire, whereon it vibrates, as on a rolling axis.

Y is a perforated table for ſupporting the axis of the pendulum.

Note,

Note, The pendulum being so adjusted, as to make two vibrations in the time that the arm F G is intended to make one turn; the pendulum being set a vibrating, the experimenter pulls by the cord Z, with sufficient force to make each half revolution of the arm to correspond with each vibration, as equal as possible, during the number of vibrations that the experiment is intended to be continued. A little practice renders it easy to give motion thereto with all the regularity that is necessary.

Specimen of a Sett of Experiments.

Radius of the sails — — —	21 inches
Length of ditto in the cloth — —	18
Breadth of ditto — — —	5,6
* Angle at the extremity — —	10 degrees
* Ditto at the greatest inclination —	25
20 turns of the sails raised the weight	11, 3 inches
Velocity of the center of the sails, in the circumference of the great circle, in a second — — — —	6 ft. 0 in.
Continuance of the experiment —	52 seconds.

N°.	Wt. in the scale.	Turns.	Product.
1	0 *lb.*	108	0
2	6	85	510
3	6½	81	526½
4	7	78	546
5	7½	73	547½ maxim^m
6	8	65	520
7	9	0	0

* In all the following experiments the angle of the sails is accounted from the plain of their motion; that is, when they stand at right angles to the axis, their angle is denoted 0°, this notation being agreeable to the language of practitioners, who call the angle so denoted, the weather of the sail; which they denominate greater or less, according to the quantity of this angle.

N. B.

N. B. The weight of the ſcale and pulley was 3 oz.; and that 1 oz. ſuſpended upon one of the radii, at $12\frac{1}{2}$ iches from the center of the axis, juſt overcame the friction ſcale and load of $7\frac{1}{2}$ lb.; and placed at $14\frac{10}{20}$ inches, overcame the ſame reſiſtances with 9 lb. in the ſcale.

Reduction of the preceding Specimen.

N°. 5. being taken for the maximum, the weight in the ſcale was 7 lb. 8 oz. which, with the weight of the ſcale and pulley 3 oz. makes 7 lb. 11 oz. equal to 123 oz.; this added to the friction of the machinery, the ſum is the whole reſiſtance *. The friction of the machinery is thus deduced: Since 20 turns of the ſails raiſed the weight 11,3 inches, with a double line, the radius of the cylinder will be .18 of an inch; but had the weight been raiſed by a ſingle line, the radius of the cylinder being half the former, *viz.* .09, the reſiſtance would have been the ſame: we ſhall therefore have this analogy; as half the radius of the cylinder, is to the length of the arm where the ſmall weight was applied; ſo is the weight applied to the arm, to a fourth weight, which is equivalent to the ſum of the whole reſiſtance together; that is, .09 : 12,5 :: 1 oz. : 139 oz.: this exceeds 123 oz. the weight in the ſcale, by 16 oz. or 1 lb. which is equivalent to the friction; and which, added to the above weight of 7 lb. 11 oz. makes 8 lb. 11 oz. = 8,69 lb. for the ſum of the whole re-

* The reſiſtance of the air is not taken into the account of reſiſtance, becauſe it is inſeparable from the application of the power.

ſiſtance; and this, multiplied by 73 turns, makes a product of 634, which may be called the repreſentative of the *effect* produced.

In like manner, if the weight 9 lb. which cauſed the ſails to reſt after being in motion, be augmented by the weight of the ſcale and its relative friction, it will become 10,37 lb. The reſult of this ſpecimen is ſet down in N°. 12. of Table III. and the reſult of every other ſett of experiments therein contained were made and reduced in the ſame manner.

TABLE III. *Containing Nineteen Setts of Experiments on Windmill-Sails of various Structures, Positions, and Quantities of Surfaces.*

The kind of sails made use of.	N°.	Angle at the extremities.	Greatest angle.	Turns of the sails unloaded.	Turns of ditto at the maxim^{m}.	Load at the maximum.	Greatest load.	Product.	Quantity of surface.	Ratio of greatest velocity to the velocity at a maximum.	Ratio of greatest load to the load at maximum.	Ratio of surface to the product.
						lb.	*lb.*		*Sq.In*			
Plain sails at an angle of 55°.	1	35°	35°	66	42	7,56	12,59	318	404	10:7	10:6	10:7,9
Plain sails weather'd according to the common practice.	2	12	12		70	6,3	7,56	441	404		10:8,3	10:10,1
	3	15	15	105	69	6,72	8,12	464	404	10:6,6	10:8,3	10:10,15
	4	18	18	96	66	7,0	9.81	462	404	10:7,	10:7,1	10:10,15
Weathered according to *Maclaurin's theorem.*	5	9	26½		66	7,0		462	404			10:11,4
	6	12	29½		70½	7,35		518	404			10:12,8
	7	15	32½		63½	8,3		527	404			10:13,
Sails weathered in the *Dutch* manner, tried in various positions.	8	0	15	120	93	4,75	5,31	442	404	10:7,7	10:8,9	10:11,
	9	3	18	120	79	7,0	8,12	553	404	10:6,6	10:8,6	10:13,7
	10	5	20		78	7,5	8,12	585	404		10:9,2	10:14,5
	11	7½	22½	113	77	8,3	9,81	639	404	10:6,8	10:8,5	10:15,8
	12	10	25	108	73	8,69	10,37	634	404	10:6,8	10:8,4	10:15,7
	13	12	27	100	66	8,41	10,94	580	404	10:6,6	10:7,7	10:14,4
Sails weathered in the *Dutch* manner, but *enlarged* towards the extremities.	14	7½	22½	123	75	10,65	12,59	799	505	10:6,1	10:8,5	10:15,8
	15	10	25	117	74	11,08	13,69	820	505	10:6,3	10:8,1	10:16,2
	16	12	27	114	66	12,09	14,23	799	505	10:5,8	10:8,4	10:15,8
	17	15	30	96	63	12,09	14,78	762	505	10:6,6	10:8,2	10:15,1
8 sails being *sectors* of *ellipses* in their best positions.	18	12	22	105	64½	16,42	27,87	1059	854	10:6,1	10:5,9	10:12,4
	19	12	22	99	64½	18,06		1165	1146	10:5,9		10:10,1
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.

Observations

Obſervations and Deductions from the preceding Experiments.

I. *Concerning the beſt Form and Poſition of Wind-mill-Sails.*

In Table III. N°. 1. is contained the reſult of a ſett of experiments upon ſails ſet at the angle which the celebrated Monſ. Parint, and ſucceeding geometricians for many years, held to be the beſt; *viz.* thoſe whoſe planes make an angle 55° nearly with the axis; the complement whereof, or angle that the plane of the ſail makes with the plane of their motion, will therefore be 35°, as ſet down in col. 2. and 3. Now if we multiply their number of turns by the weight they lifted, when working to the greateſt advantage, as ſet down in columns 5. and 6. and compare this product (col. 8.) with the other products contained in the ſame column, inſtead of being the greateſt, it turns out the leaſt of all the reſt. But if we ſet the angle of the ſame planes at ſomewhat leſs than half the former, or at any angle from 15° to 18°, as in N°. 3. and 4. that is, from 72° to 75° with the axis, the product will be increaſed in the ratio of 31 : 45; and this is the angle moſt commonly made uſe of by practitioners, when the ſurfaces of the ſails are planes.

If nothing more was intended than to determine the moſt efficacious angle to make a mill acquire motion from a ſtate of reſt, or to prevent it from paſſing into reſt from a ſtate of motion, we ſhall find the poſition of N°. 1. the beſt; for if we conſult col. 7. which contains the leaſt weights, that would make the ſails paſs from motion to reſt, we ſhall find that of N°. 1.

(relative to the quantity of cloth) the greateſt of all. But if the ſails are intended, with given dimenſions, to produce the greateſt effect poſſible in a given time, we muſt intirely reject thoſe of N°. 1. and, *if we are confined to the uſe of planes, conform ourſelves to ſome angle between* N°. 3. *and* 4. *that is, not leſs than* 72°, *or greater than* 75°, *with the axis.*

The late celebrated Mr. Maclaurin has judiciouſly diſtinguiſhed between the action of the wind upon a ſail at reſt, and a ſail in motion; and, in conſequence, as the motion is more rapid near the extremities than towards the center, that the angle of the different parts of the ſail, as they recede from the center, ſhould be varied. For this purpoſe he has furniſhed us with the following theorem *. "Suppoſe the velocity "of the wind to be repreſented by *a*, and the velo- "city of any given part of the ſail to be denoted by "*c*; then the effort of the wind upon that part of "the ſail will be greateſt when the tangent of the "angle, in which the wind ſtrikes it, is to radius as "$\sqrt{2 + \frac{9cc}{4aa}} + \frac{3c}{2a}$ to 1." This theorem then aſſigns the law, by which the angle is to be varied according to the velocity of each part of the ſail to the wind: but as it is left undetermined what velocity any one given part of the ſail ought to have in reſpect to the wind, the angle that any one part of the ſail ought to have, is left undetermined alſo; ſo that we are ſtill at a loſs for the proper *data* to apply the theorem. However, being willing to avail myſelf thereof, and conſidering that any angle from 15° to 18° was beſt ſuited to a plane, and of conſequence the beſt

* Maclaurin's account of Sir Iſaac Newton's philoſophical diſcoveries, p. 176, art. 29.

mean

mean angle, I made the ſail, at the middle diſtance between the center and the extremity, to ſtand at an angle of 15° 41 with the plane of the motion; in which caſe the velocity of that part of the ſail, when loaded to a *maximum*, would be equal to that of the wind, or $c = a$. This being determined, the reſt were inclined according to the theorem, as follows:

Parts of the radius from the center.		Angle with the axis.	Angle of weather.	
$\frac{1}{6}$	$c = \frac{1}{3}a$	63° 26′	26° 34′	
$\frac{2}{6}$	$c = \frac{2}{3}a$	69 54	20 6	
$\frac{1}{2}$	$c = a$	74 19	15 41	middle
$\frac{2}{3}$	$c = 1\frac{1}{3}a$	77 20	12 40	
$\frac{5}{6}$	$c = 1\frac{3}{2}a$	79 27	10 33	
1	$c = 2a$	81 0	9 0	extremity.

The reſult hereof was according to N°. 5. being nearly the ſame as the plane ſails, in their beſt poſition: but being turned round in their ſockets, ſo that every part of each ſail ſtood at an angle of 3°, and afterwards of 6°, greater than before, that is, their extremities being moved from 9° to 12° and 15°, the products were advanced to 518 and 527 reſpectively. Now from the ſmall difference between thoſe two products, we may conclude, that they were nearly in their beſt poſition, according to N°. 7. or ſome angle between that and N°. 6: but from theſe, as well as the plane ſails and others, we may alſo conclude, that *a variation in the angle of a degree or two makes very little difference in the effect, when the angle is near upon the beſt.*

It is to be obſerved, that a ſail inclined by the preceding rule will expoſe a convex ſurface to the wind; whereas the Dutch, and all our modern

 mill-

mill-builders, tho' they make the angle to diminiſh, in receding from the center towards the extremity, yet conſtantly do it in ſuch manner, as that the ſurface of the ſail may be concave towards the wind. In this manner the ſails made uſe of in N°. 8, 9, 10, 11, 12, and 13. were conſtructed; the middle of the ſail making an angle with the extreme bar of 12°; and the greateſt angle (which was about $\frac{1}{3}$ of the radius from the centre) of 15° therewith. Thoſe ſails being tried in various poſitions, the beſt appears to be that of N°. 11. where the extremities ſtood at an angle of 7°$\frac{1}{2}$ with the plane of motion, the product being 639: greater than that of thoſe made by the theorem in the ratio of 9 : 11, and double to that of N°. 1.; and this was the greateſt product that could be procured without an augmentation of ſurface. Hence it appears, that *when the wind falls upon a concave ſurface, it is an advantage to the power of the whole, tho' every part, taken ſeparately, ſhould not be diſpoſed to the beſt advantage* *.

Having thus obtained the beſt poſition of the ſails, or manner of weathering, as it is called by workmen, the next point was to try what advantage could be made

* By ſeveral trials in large I have found the following angles to anſwer as well as any. The radius is ſuppoſed to be divided into 6 parts and 1-6th, reckoning from the center, is called 1, the extremity being denoted 6.

N°.	Angle with the axis.	Angle with the plane of motion.
1	72°	18°
2	71	19
3	72	18 middle.
4	74	16
5	77$\frac{1}{2}$	12$\frac{1}{2}$
6	83	7 extremity.

made by an addition of ſurface upon the ſame radius. For this purpoſe, the ſails made uſe of had the ſame weather as thoſe N°. 8. to 13, with an addition to the leading ſide of each of a triangular cloth, whoſe height was equal to the height of the ſail, and whoſe baſe was equal to half the breadth: of conſequence the increaſe of ſurface upon the whole was one fourth part, or as 4 : 5. Thoſe ſails, by being turned round in their ſockets, were tried in four different poſitions, ſpecified in N°. 14, 15, 16, and 17; from whence it appears, that the beſt was when every part of the ſail made a greater angle by 2° ½, with the plane of the motion, than thoſe without the addition, as appears by N°. 15. the product being 820: this exceeds 639 more than in the ratio of 4 : 5, or that of the increaſe of cloth. Hence it appears, that *a broader ſail requires a greater angle*; and *that when the ſail is broader at the extremity, than near the center, this ſhape is more advantageous than that of a parallelogram* *.

Many have imagined, that the more ſail, the greater the advantage, and have therefore propoſed to fill up the whole area: and by making each ſail a ſector of an ellipſis, according to Monſieur Parint, to intercept the whole cylinder of wind, and thereby to produce the greateſt effect poſſible.

* The figure and proportion of the enlarged ſails, which I have found beſt to anſwer in large, are repreſented in the figure, Plate VI. where the extreme bar is 1-3d of the radius (or whip, as it is called by the workmen), and is divided by the whip in the proportion of 3 to 5. The triangular or leading ſail is covered with board from the point downwards 1-3d of its height, the reſt with cloth as uſual. The angles of weather in the preceding note are beſt for the enlarged ſails alſo; for in practice it is found, that the ſails had better have too little than too much weather.

We have therefore proceeded to inquire, how far the effect could be increafed by a further enlargement of the furface, upon the fame radius of which N°. 18 and 19 are fpecimens. The furfaces indeed were not made planes, and fet at an angle of 35°, as Parint propofed; becaufe, from N°. 1. we learn, that this pofition has nothing to do, when we intend them to work to the greateft advantage. We therefore gave them fuch an angle as the preceding experiments indicated for fuch fort of fails, *viz.* 12° at the extremity, and 22° for the greateft weather. By N°. 18 we have the product 1059, greater than N°. 15. in the ratio of 7 : 9; but then the augmentation of cloth is almoft 7 : 12. By N°. 19. we have the product 1165, that is greater than N°. 15. as 7 : 10; but the augmentation of cloth is nearly as 7 : 16; confequently had the fame quantity of cloth as in N°. 18. been difpofed in a figure fimilar to that of N°. 15, inftead of the product 1059, we fhould have had the product 1386; and in N°. 19, inftead of the product 1165, we fhould have had a product of 1860; as will be further made appear in the courfe of the following deductions. Hence it appears, that beyond a certain degree, the more the area is crowded with fail, the lefs effect is produced in proportion to the furface: and by purfuing the experiments ftill further, I found, that tho' in N°. 19. the furface of all the fails together were not more than 7-8ths of the circular area containing them, yet a further addition rather diminifhed than increafed the effect. *So that when the whole cylinder of wind is intercepted, it does not then produce the greateft effect for want of proper interftices to efcape.*

It

It is certainly desirable, that the sails of windmills should be as short as possible; but at the same time it is equally desirable, that the quantity of cloth should be the least that may be, to avoid damage by sudden squalls of wind. The best structure, therefore, for large mills, is that where the quantity of cloth is the greatest, in a given circle, that can be: on this condition, that the effect holds out in proportion to the quantity of cloth; for otherwise the effect can be augmented in a given degree by a lesser increase of cloth upon a larger radius, than would be required, if the cloth was increased upon the same radius. The most useful figure therefore for practice, is that of N°. 9. or 10. as has been experienced upon several mills in large.

TABLE IV. *Containing the Result of six Setts of Experiments, made for determining the Difference of Effect, according to the different Velocity of the Wind.*

N. B. The sails were of the same size and kind as those of N° 10, 11, and 12, Tab. IV. Continuance of the Experiment one minute.

N°	Angle at the extremity.	Velocity of the wind in a second.	Turns of the sails unloaded.	Turns of the sails at maximum.	Load at the maximum.	Greatest load.	Product.	Maximum load for the half velocity.	Turns of the sails therewith.	Product of lesser load and greater velocity.	Ratio of the two products.	Ratio of the greatest velocity to the velocity at a maxim^{m}.	Ratio of the greatest load to the load at a maximum.
		l. m.			*lb.*	*lb.*							
1	5°	4 4½	96	66	4,47	5,37	295	—	—	—	— —	10 : 6,9	10 : 8,3
2	5	8 9	207	122	16,42	18,06	2003	4,47	180	805	10 : 27,3	10 : 5,9	10 : 9,1
3	7½	4 4½	—	65	4,62	—	300	—	—	—	— —	—	—
4	7½	8 9	—	130	17,52	—	2278	4,62	180	832	10 : 27,8	—	—
5	10	4 4½	91	61	5,03	5,87	307	—	—	—	— —	10 : 6,7	10 : 8,5
6	10	8 9	178	110	18,61	21,34	2047	5,03	158	795	10 : 26,	10 : 6,2	10 : 8,7
1	2	3	4	5	6	7	8	9	10	11	12	13	14

II. *Concerning the ratio between the velotity of windmill ſails unloaded, and their velocity when loaded to a maximum.*

Thoſe ratio's, as they turned out in experiments upon different kinds of ſails, and with different inclinations (the velocity of the wind being the ſame) are contained in column 10 of tab. III. where the extremes differ from the ratio of 10 : 7,7 to that of 10 : 5,8; but *the moſt general ratio of the whole will be nearly as* 3 : 2. This ratio alſo agrees ſufficiently near with experiments where the velocity of the wind was different, as in thoſe contained in tab. IV. col. 13. in which the ratio's differ from 10 : 6,9 to that of 10 : 5,9. However, it appears in general, that where the power is greater, whether by an enlargement of ſurface, or a greater velocity of the wind, that the ſecond term of the ratio is leſs.

III. *Concerning the ratio between the greateſt load that the ſails will bear without ſtopping, or what is nearly the ſame thing, between the leaſt load that will ſtop the ſails, and the load at the maximum.*

Thoſe ratio's for different kinds of ſails and inclinations, are collected in col. 11. tab. III. where the extremes differ from the ratio of 10 : 6 to that of 10 : 9,2; but taking in thoſe ſetts of experiments only, where the ſails reſpectively anſwered beſt, *the ratio's will be confined between that of* 10 : 8 *and of* 10 : 9; *and at a medium about* 10 : 8,3 *or of* 6 : 5. This ratio alſo agrees nearly with thoſe in col. 14 of tab. IV. However it appears, upon the whole, that in thoſe inſtances, where the angle of the ſails or quan-

quantity of cloth were greateſt, that the ſecond term of the ratio was leſs.

IV. *Concerning the effects of ſails, according to the different velocity of the wind.*

Maxim 1. *The velocity of windmill ſails, whether unloaded, or loaded ſo as to produce a maximum, is nearly as the velocity of the wind, their ſhape and poſition being the ſame.*

This appears by comparing together the reſpective numbers of columns 4 and 5, tab. IV. wherein thoſe of numbers 2, 4, and 6, ought to be double of numbers 1, 3, and 5: but as the deviation is no-where greater than what may be imputed to the inaccuracy of the experiments themſelves, and hold good exactly in numbers 3 and 4; which ſetts were deduced from the medium of a number of experiments, carefully repeated the ſame day, and on that account are moſt to be depended upon; we may therefore conclude the maxim true.

Maxim 2. *The load at the maximum is nearly, but ſomewhat leſs than, as the ſquare of the velocity of the wind, the ſhape and poſition of the ſails being the ſame.*

This appears by comparing together the numbers in col. 6. tab. IV. wherein thoſe of numbers 2, 4, and 6 (as the velocity is double), ought to be quadruple of thoſe of numbers 1, 3, and 5; inſtead of which they fall ſhort, number 2 by $\frac{1}{14}$, number 4 by $\frac{1}{19}$, and number 6 by $\frac{1}{13}$ part of the whole. The greateſt of thoſe deviations is not more conſiderable than might be imputed to the unavoidable errors

errors in making the experiments: but as thoſe experiments, as well as thoſe of the greateſt load, all deviate the ſame way; and alſo coincide with ſome experiments communicated to me by Mr. Rouſe upon the reſiſtance of planes; I am led to ſuppoſe a ſmall deviation, whereby the load falls ſhort of the ſquares of the velocity; and ſince the experiments N° 3 and 4. are moſt to be depended upon, we muſt conclude, that when the velocity is double, the load falls ſhort of its due proportion by $\frac{1}{19}$, or, for the ſake of a round number, by about $\frac{1}{20}$ part of the whole.

Maxim 3d. *The effects of the ſame ſails at a maximum are nearly, but ſomewhat leſs than, as the cubes of the velocity of the wind.*

It has already been proved, Maxim 1ſt, that the velocity of ſails at the *maximum*, is nearly as the velocity of the wind; and by Maxim 2d, that the load at the *maximum* is nearly as the ſquare of the ſame velocity: if thoſe two maximums would hold preciſely, it would be a conſequence that the effect would be in a triplicate ratio thereof: how this agrees with experiment will appear by comparing together the products in col. 8. of tab. 4. wherein thoſe of N° 2. 4. and 6. (the velocity of the wind being double) ought to be octuple of thoſe of N° 1. 3. and 5. inſtead of which they fall ſhort, N° 2. by $\frac{1}{7}$ N° 4. by $\frac{1}{20}$, and N° 6. by $\frac{1}{6}$ part of the whole. Now, if we rely on N° 3. and 4. as the turns of the ſails are as the velocity of the wind; and ſince the load of the maximum falls ſhort of the ſquare of the velocity by about $\frac{1}{20}$ part of the whole: the product

made by the multiplication of the turns into the load, muſt alſo fall ſhort of the triplicate ratio by about $\frac{1}{20}$ part of the whole product.

Maxim 4th. *The load of the ſame ſails at the maximum is nearly as the ſquares, and their effect as the cubes, of their number of turns in a given time.*

This maxim may be eſteemed a conſequence of the three preceding; for if the turns of the ſails are as the velocity of the wind, whatever quantities are in any given ratio of the velocity of the wind, will be in the ſame given ratio of the turns of the ſails: and therefore, if the load at the *maximum* is as the ſquare, or the effect as the cube, of the velocity of the wind, wanting $\frac{1}{20}$ part when the velocity is double; the load at the *maximum* will alſo be as the ſquare, and the effect as the cube, of the number of turns of the ſails in a given time, wanting in like manner $\frac{1}{20}$ part when the number of turns are double in the ſame time. In the preſent caſe, if we compare the loads at the *maximum* col. 6. with the ſquares of the number of turns col. 5. of N° 1 and 2. 5 and 6. or the products of the ſame numbers col. 8. with the cubes of the number of turns col. 5. inſtead of falling ſhort, as N° 3 and 4. they exceed thoſe ratios: but as the ſetts of experiments N° 1 and 2. 5 and 6. are not to be eſteemed of equal authority with thoſe of N° 3 and 4. we muſt not rely upon them further than to obſerve, that *in comparing the groſs effects of large machines, the direct proportion of the ſquares and cubes reſpectively, will hold as near as the effects themſelves can be obſerved*; and therefore

fore be ſufficient for practical eſtimation, without any allowance.

Maxim 5th. *When ſails are loaded ſo as to produce a maximum at a given velocity, and the velocity of the wind increaſes, the load continuing the ſame; 1ſtly, The increaſe of effect, when the increaſe of the velocity of the wind is ſmall, will be nearly as the ſquares of thoſe velocities: 2dly, When the velocity of the wind is double, the effects will be nearly as* 10 : $27\frac{1}{2}$: *But, 3dly, When the velocities compared, are more than double of that where the given load produces a maximum, the effects increaſe nearly in a ſimple ratio of the velocity of the wind.*

It has already been proved, maxim 1ſt and 2d, that when the velocity of the wind is increaſed, the turns of the ſails will increaſe in the ſame proportion, even when oppoſed by a load as the ſquare of the velocity; and therefore if wanting the oppoſition of an increaſe of load, as the ſquare of the velocity, the turns of the ſails will again be increaſed in a ſimple ratio of the velocity of the wind on that account alſo; that is, the load continuing the ſame, the turns of the ſails in a given time will be as the ſquare of the velocity of the wind; and the effect, being in this caſe as the turns of the ſails, will be as the ſquare of the velocity of the wind alſo; but this muſt be underſtood only of the firſt increments of the velocity of the wind: for,

2dly, As the ſails will never acquire above a given velocity in relation to the wind, tho' the load was diminiſhed to nothing; when the load continues the

ſame,

ſame, the more the velocity of the wind increaſes (tho' the effect will continue to increaſe) yet the more it will fall ſhort of the ſquare of the velocity of the wind; ſo that when the velocity of the wind is double, the increaſe of effect, inſtead of being as 1 : 4, according to the ſquares, it turns out as 10 : 27½, as thus appears. In tab. 4. col. 9. the loads of N° 2, 4, and 6. are the ſame as the maximum loads in col. 6. of N° 1, 3, and 5. The number of turns of the ſails with thoſe loads, when the velocity of the wind is double, are ſet down in col. 10. and the products of their multiplification in col. 11: thoſe being compared with the products of N° 1, 3, and 5. col. 8. furniſh the ratios ſet down in col. 12. which at a medium (due regard being had to N° 3 and 4.) will be nearly as 10 : 27½. 3dly. The load continuing the ſame, grows more and more inconſiderable, reſpecting the power of the wind as it increaſes in velocity; ſo that the turns of the ſails grow nearer and nearer a coincidence with their turns unloaded; that is, nearer and nearer to the ſimple ratio of the velocity of the wind. When the velocity of the wind is double, the turns of the ſails, when loaded to a maximum, will be double alſo; but, *unloaded*, will be no more than triple, by deduction 2d: and therefore the product could not have increaſed beyond the ratio of 10:30 (inſtead of 10 : 27½) even ſuppoſing the ſails not to have been retarded at all by carrying the maximum load for the half velocity. Hence we ſee, that when the velocity of the wind exceeds the double of that, where a conſtant load produces a maximum, that the increaſe of effect, which follows the increaſe of the velocity of the ſails, will be nearly as the velocity

city of the wind, and ultimately in that ratio precifely. Hence alfo we fee that windmills, fuch as the different fpecies for raifing water for drainage, &c. lofe much of their full effect, when acting againft one invariable oppofition.

V. *Concerning the effects of fails of different magnitudes, the ftructure and pofition being fimilar, and the velocity of the wind the fame.*

Maxim 6. *In fails of a fimilar figure and pofition, the number of turns in a given time will be reciprocally as the radius or length of the fail.*

The extreme bar having the fame inclination to the plain of its motion, and to the wind; its velocity at a *maximum* will always be in a given ratio to the velocity of the wind; and therefore, whatever be the radius, the abfolute velocity of the extremity of the fail will be the fame: and this will hold good refpecting any other bar, whofe inclination is the fame, at a proportionable diftance from the center; it therefore follows, that the extremity of all fimilar fails, with the fame wind, will have the fame abfolute velocity; and therefore take a fpace of time to perform one revolution in proportion to the radius; or, which is the fame thing, the number of revolutions in the fame given time, will be reciprocally as the length of the fail.

Maxim 7. *The load at a maximum that fails of a fimilar figure and pofition will overcome, at a given diftance from the center of motion, will be as the cube of the radius.*

Geo-

Geometry informs us, that in ſimilar figures the ſurfaces are as the ſquares of their ſimilar ſides; of conſequence the quantity of cloth will be as the ſquare of the radius: alſo in ſimilar figures and poſitions, the impulſe of the wind, upon every ſimilar ſection of the cloth, will be in proportion to the ſurface of that ſection; and conſequently, the impulſe of the wind upon the whole, will be as the ſurface of the whole: but as the diſtance of every ſimilar ſection, from the center of motion, will be as the radius; the diſtance of the center of power of the whole, from the center of motion, will be as the radius alſo; that is, the lever by which the power acts, will be as the radius: as therefore the impulſe of the wind, reſpecting the quantity of cloth, is as the ſquare of the radius, and the lever, by which it acts, as the radius ſimply; it follows, that the load which the ſails will overcome, at a given diſtance from the center, will be as the cube of the radius.

Maxim 8. *The effect of ſails of ſimilar figure and poſition, are as the ſquare of the radius.*

By maxim 6. it is proved, that the number of revolutions made in a given time, are as the radius inverſely. Under maxim 7. it appears, that the length of the lever, by which the power acts, is as the radius directly; therefore theſe equal and oppoſite ratios deſtroy one another: but as in ſimilar figures the quantity of cloth is as the ſquare of the radius, and the action of the wind is in proportion to the quantity of cloth, as alſo appears under maxim 7; it follows that the effect is as the ſquare of the radius.

COROL. 1. Hence it follows, that augmenting the length of the ſail, without augmenting the quantity of cloth, does is not increaſe the power; becauſe what is gained by the length of the lever, is loſt by the ſlowneſs of the rotation.

COROL. 2. If ſails are increaſed in length, the breadth remaining the ſame, the effect will be as the radius.

VI. *Concerning the velocity of the extremities of windmill ſails, in reſpect to the velocity of the wind.*

Maxim 9. *The velocity of the extremities of* Dutch *ſails, as well as of the enlarged ſails, in all their uſual poſitions when unloaded, or even loaded to a maximum, are conſiderably quicker than the velocity of the wind.*

The *Dutch* ſails unloaded, as in Tab. 3. No 8. made 120 revolutions in 52″: the diameter of the ſails being 3 feet 6 inches, the velocity of their extremities will be 25,4 feet in a ſecond; but the velocity of the wind producing it, being 6 feet in the ſame time, we ſhall have 6 : 25,4 :: 1 : 4,2; in this caſe therefore, the velocity of their extremities was 4,2 times greater than that of the wind. In like manner, the relative velocity of the wind, to the extremities of the ſame ſails, when loaded to a *maximum*, making then 93 turns in 52″, will be found to be as 1 : 3,3; or 3,3 times quicker than that of the wind.

The

The following table contains 6 examples of *Dutch* ſails, and 4 examples of the enlarged ſails, in different poſitions, but with the conſtant velocity of the wind of 6 feet in a ſecond, from table 3: and alſo 6 examples of *Dutch* ſails in different poſitions, with different velocities of the wind, from table 4.

TABLE V. *containing the ratio of the velocity of the extremities of windmill ſails to the velocity of the wind.*

N°	N° of Tab. III and IV.	Angle at the Extremity.	Velocity of the wind in a ſecond.	Ratio of the velocity of the wind and extremities of the ſails. unloaded.	loaded.	
1	8	0°	$6^{f}\ 0^{in}$	1 : 4,2	1 : 3,3	From Table III.
2	9	3	6 0	1 : 4,2	1 : 2,8	
3	10	5	6 0	— — —	1 : 2,75	
4	11	7½	6 0	1 : 4,	1 : 2,7	
5	12	10	6 0	1 : 3,8	1 : 2,6	
6	13	12	6 0	1 : 3,5	1 : 2,3	
7	14	7½	6 0	1 : 4,3	1 : 2,6	
8	15	10	6 0	1 : 4,1	1 : 2,6	
9	16	12	6 0	1 : 4,	1 : 2,3	
10	17	15	6 0	1 : 3,35	1 : 2,2	
11	1	5	4 4½	1 : 4,	1 2,8	From Tab. IV.
12	2	5	8 9	1 : 4,3	1 2,6	
13	3	7½	4 4½	— — —	1 : 2,8	
14	4	7½	8 9	— — —	1 : 2,7	
15	5	10	4 4½	1 : 3,8	1 2,6	
16	6	10	8 9	1 : 3,4	1 : 2 3	
1	2	3	4	5	6	

It appears from the preceding collection of examples, that when the extremities of the *Dutch* ſails are parallel to the plane of motion, or at right angles to the wind, and to the axis, as they are made according to the common practice in *England*, that their velocity, unloaded, is above 4 times, and loaded to a *maximum*, above 3 times greater than that of the wind: but that when the Dutch ſails, or enlarged ſails, are in their beſt poſitions, their velocity unloaded is 4 times, and loaded to a *maximum*, at a medium the Dutch ſails are 2,7, and the enlarged ſails 2,6 times greater than the velocity of the wind. Hence we are furniſhed with a method of knowing the velocity of the wind, from obſerving the velocity of the windmill ſails; for knowing the radius, and the number of turns in a minute, we ſhall have the velocity of the extremities; which, divided by the following diviſors, will give the velocity of the wind.

Dutch ſails in their common poſition	unloaded	4.2
	loaded —	3.3
Dutch ſails in their beſt poſition --	unloaded	4.0
	loaded —	2.7
Enlarged ſails in their beſt poſition	unloaded	4.0
	loaded —	2.6

From the above diviſors there ariſes the following compendiums; ſuppoſing the radius to be 30 feet, which is the moſt uſual length in this country, and the mill to be loaded to a *maximum*, as is uſually the caſe with corn mills; for every 3 *turns in a minute, of the Dutch ſails in their common poſition, the wind will move at the rate of* 2 *miles an hour*; for every 5 *turns in a minute, of the Dutch ſails in their beſt*

position, the wind moves 4 miles an hour; and for every 6 *turns in a minute, of the enlarged sails in their best position, the wind will move 5 miles an hour.*

The following table, which was communicated to me by my friend Mr. Rouse, and which appears to have been constructed with great care, from a considerable number of facts and experiments, and which having relation to the subject of this article; I here insert it as he sent it to me: but at the same time must observe, that the evidence for those numbers where the velocity of the wind exceeds 50 miles an hour, do not seem of equal authority with those of 50 miles an hour and under. It is also to be observed, that the numbers in col. 3. are calculated according to the square of the velocity of the wind, which, in moderate velocities, from what has been before observed, will hold very nearly.

TABLE VI. *containing the velocity and force of wind, according to their common appellations.*

Velocity of the Wind. Miles in one Hour.	Velocity of the Wind. Feet in one fecond.	Perpendicular force on one foot area in pounds averdupois.	Common appellations of the force of winds.
1	1,47	,005	Hardly perceptible.
2	2,93	,020	Juft perceptible.
3	4,40	,044	
4	5,87	,079	Gentle pleafant wind.
5	7,33	,123	
10	14,67	,492	Pleafant brifk gale.
15	22,00	1,107	
20	29,34	1,968	Very brifk.
25	36,67	3,075	
30	44,01	4,429	High winds.
35	51,34	6,027	
40	58,68	7,873	Very high.
45	66,01	9,963	
50	73,35	12,300	A ftorm or tempeft.
60	88,02	17,715	A great ftorm.
80	117,36	31,490	An hurricane.
100	146,70	49,200	An hurircane that tears up trees, carries buildings before it, &c.
1	2	3	

VII. *concerning the abfolute effect, produced by a given velocity of the wind, upon fails of a given magnitude and conftruction.*

It has been obferved by practitioners, that in mills with Dutch fails in the common pofition, that when they make about 13 turns in a minute, they then work

work at a mean rate: that is, by the compendiums in the laſt article, when the velocity of the wind is $8\frac{2}{3}$ miles an hour, or $12\frac{2}{3}$ feet in a ſecond; which, in common phraſe, would be called a *freſh gale*.

The experiments ſet down in Tab. IV. No 4. were tried with a wind, whoſe velocity was $8\frac{3}{4}$ feet in a ſecond; conſequently had thoſe experiments been tried with a wind, whoſe velocity was $12^{\overline{3}}$ feet in a ſecond, the effect, by maxim 3d, would have been 3 times greater; becauſe the cube of $12\frac{2}{3}$ is 3 times greater than that of $8\frac{3}{4}$.

From Tab. IV. No 4. we find, that the ſails, when the velocity of the wind was $8\frac{3}{4}$ feet in a ſecond, made 130 revolutions in a minute, with a load of 17,25 ℔. From the meaſures of the machine, preceding the ſpecimen of a ſett of experiments, we find, that 20 revolutions of the ſails raiſed the ſcale and weight 11,3 inches: 130 revolutions will therefore raiſe the ſcale 73,45 inches, which, multiplied by 17,52 ℔, makes a product of 1287, for the effect of the Dutch ſails in their beſt poſition; that is, when the velocity of the wind is $8\frac{3}{4}$ feet in a ſecond: this product therefore multiplied by three, will give 3861 for the effect of the ſame ſails, when the velocity of the wind is $12\frac{2}{3}$ feet in a ſecond.

Deſaguliers makes the utmoſt power of a man, when working ſo as to be able to hold it for ſome hours, to be equal to that of raiſing an hogſhead of water 10 feet high in a minute. Now, an hogſhead conſiſting of 63 ale gallons, being reduced into pounds averdupois, and the height into inches; the product made by multiplying thoſe two numbers will be 76800; which is 19 times greater than the

pro-

product of the ſails laſt-mentioned, at 12½ feet in a ſecond: therefore, by maxim 8th, if we multiply the ſquare root of 19, that is 4,46, by 21 inches, the length of the ſail producing the effect 3861, we ſhall have 93,66 inches, or 7 feet 9⅔ inches for the radius of a Dutch ſail in its beſt poſition, whoſe mean power ſhall be equal to that of a man: but if they are in their common poſition, their length muſt be increaſed in the ratio of the ſquare root of 442 to that of 639, as thus appears;

The ratio of the *maximum* products of N° 8 and 11. Tab. III. are as 442 : 639; but by maxim 8, the effects of ſails of different radii are as the ſquare of the radii; conſequently the the ſquare roots of the products or effects, are as the radii ſimply; and therefore as the ſquare root of 442 is to that of 639; ſo is 93,66 to 112,66; or 9 feet 4⅔ inches.

If the ſails are of the enlarged kind, then from Tab. III. N° 11 and 15. we ſhall have the ſquare root of 820 to that of 639::93,66 : 82,8 inches, or 6 feet 10¾ inches: ſo that in round numbers we ſhall have the radius of a ſail, of a ſimilar figure to their reſpective models, whoſe mean power ſhall be equal to that of a man;

The Dutch ſails in their common poſition	9½ feet.
The Dutch ſails in their beſt poſition ——	8
The enlarged ſails in their beſt poſition —	7

Suppoſe now the radius of a ſail to be 30 feet, and to be conſtructed upon the model of the enlarged ſails, N° 14 or 15. Tab. III. dividing 30 by 7 we, ſhall have 4,28, the ſquare of which is 18,3; and this, according to maxim 7, will be the relative

3 power

power of a sail of 30 feet, to one of 7 feet; that is, when working at a mean rate, the 30 feet sail will be equal to the power of 18,3 men, or of $3\frac{2}{3}$ horses; reckoning 5 men to a horse: whereas the effect of the common Dutch sails, of the same length, being less in the proportion of 820:442, will be scarce equal to the power of 10 men, or of 2 horses.

That these computations are not merely speculative, but will nearly hold good when applied to works in large, I have had an opportunity of verifying: for in a mill with the enlarged sails of 30 feet, applied to the crushing of rape seed, by means of two runners upon the edge, for making oil; I observed, that when the sails made 11 turns in a minute, in which case the velocity of the wind was about 13 feet in a second, according to article 6th, that the runners then made 7 turns in a minute: whereas 2 horses, applied to the same 2 runners, scarcely worked them at the rate of $3\frac{1}{2}$ turns in the same time. Lastly, with regard to the real superiority of the enlarged sails, above the Dutch sails as commonly made, it has sufficiently appeared, not only in those cases where they have been applied to new mills, but where they have been substituted in the place of the others.

VIII. *Concerning horizontal windmills and water-wheels, with oblique vanes.*

Observations upon the effects of common windmills with oblique vanes, have led many to imagine, that could the vanes be brought to receive the direct impulse, like a ship sailing before the wind, it would

be

be a very great improvement in point of power: while others attending to the extraordinary and even unexpected effects of oblique vanes, have been led to imagine, that oblique vanes applied to water-mills, would as much exceed the common water wheels, as the vertical windmills are found to have exceeded all attempts towards an horizontal one. Both these notions, but especially the first, have so plausible an appearance, that of late years there has seldom been wanting those, who have assiduously employed themselves to bring to bear designs of this kind: it may not therefore be unacceptable to endeavour to set this matter in a clear light.

PLATE VI. fig 2d. Let A B be the section of a plain, upon which let the wind blow in the direction C D, with such a velocity as to describe a given space B E, in a given time (suppose 1 second); and let A B be moved parallel to itself, in the direction C D. Now, if the plane A B moves with the same velocity as the wind; that is, if the point B moves thro' the space B E in the same time that a particle of air would move thro' the same space; it is plain that, in this case, there can be no pressure or impulse of the wind upon the plane: but if the plane moves slower than the wind, in the same direction, so that the point B may move to F, while a particle of air, setting out from B at the same instant, would move to E, then B F will express the velocity of the plane; and the relative velocity of the wind and plane will be expressed by the line F E. Let the ratio of F E to B E be given (suppose 2 : 3.); let the line A B represent the impulse of the wind upon the plane A B, when acting with its whole velocity B E; but,

 when

when acting with its relative velocity F E, let its impulse be denoted by some aliquot part of A B, as for instance $\frac{4}{9}$ A B: then will $\frac{4}{9}$ of the parallelogram A F represent the mechanical power of the plane; that is, $\frac{4}{9}$ A B $\times$ $\frac{1}{3}$ B E.

2dly, Let I N be the section of a plane, inclined in such a manner, that the base I K of the rectangle triangle I K N may be equal to A B; and the perpendicular N K=B E; let the plane I N be struck by the wind, in the direction L M, perpendicular to I K: then, according to the known rules of oblique forces, the impulse of the wind upon the plain I N, tending to move it according to the direction L M, or N K, will be denoted by the base I K; and that part of the impulse, tending to move it according to the direction I K, will be expressed by the perpendicular N K. Let the plane I N be moveable in the direction of I K only; that is, the point I in the direction of I K, and the point N in the direction N Q, parallel thereto. Now it is evident, that if the point I moves thro' the line I K, while a particle of air, setting forwards at the same time from the point N, moves thro' the line N K, they will both arrive at the point K at the same time; and consequently, in this case also, there can be no pressure or impulse of the particle of the air upon the plane I N. Now let I O be to I K as B F to B E; and let the plane I N move at such a rate, that the point I may arrive at O, and acquire the position I Q, in the same time that a particle of wind would move thro' the space N K: as O Q is parallel to I N; (by the properties of similar triangles) it will cut N K in the point P, in such a manner, that N P=B F, and P K=F E: hence it

appears,

appears, that the plane I N, by acquiring the poſition O Q, withdraws itſelf from the action of the wind, by the ſame ſpace N P, that the plane A B does by acquiring the poſition F G; and conſequently, from the equality of P K to F E, the relative impulſe of the wind P K, upon the plane O Q, will be equal to the relative impulſe of the wind F E, upon the plane F G: and ſince the impulſe of the wind upon A B, with the relative velocity F E, in the direction B E, is repreſented by $\frac{4}{9}$ A B; the relative impulſe of the wind upon the plane I N, in the direction N K, will in like manner be repreſented by $\frac{4}{9}$ I K; and the impulſe of the wind upon the plane I N, with the relative velocity P K, in the direction I K, will be repreſented by $\frac{4}{9}$ N K: and conſequently the mechanical power of the plane I N, in the direction I K, will be $\frac{4}{9}$ the parallellogram I Q: that is $\frac{1}{3}$ I K $\times$ $\frac{4}{9}$ N K: that is, from the equality of I K $=$ A B and N K $=$ B E, we ſhall have $\frac{4}{9}$ I Q $=\frac{1}{3}$ A B $\times$ $\frac{4}{9}$ B E $=\frac{4}{9}$ A B $\times$ $\frac{1}{3}$ B E $=\frac{4}{9}$ of the area of the parallellogram A F. Hence we deduce this

GENERAL PROPOSITION,

That all planes, however ſituated, that intercept the ſame ſection of the wind, and having the ſame relative velocity, in regard to the wind, when reduced into the ſame direction, have equal powers to produce mechanical effects.

For what is loſt by the obliquity of the impulſe, is gained by the velocity of the motion.

Hence it appears, that an oblique ſail is under no diſadvantage in reſpect of power, compared with a direct one; except what ariſes from a diminution of

its breadth, in reſpect to the ſection of the wind: the breadth I N being by obliquity reduced to I K.

The diſadvantage of horizontal windmills therefore does not conſiſt in this; that each ſail, when directly expoſed to the wind, is capable of a leſs power, than an oblique one of the ſame dimenſions; but that in an horizontal windmill, little more than one ſail can be acting at once: whereas in the common windmill, all the four act together: and therefore, ſuppoſing each vane of an horizontal windmill, of the ſame dimenſions as each vane of the vertical, it is manifeſt the power of a vertical mill with four ſails, will be four times greater than the power of the horizontal one, let its number of vanes be what it will: this diſadvantage ariſes from the nature of the thing; but if we conſider the further diſadvantage, that ariſes from the difficulty of getting the ſails back again againſt the wind, &c. we need not wonder if this kind of mill is in reality found to have not above $\frac{1}{8}$ or $\frac{1}{10}$ of the power of the common ſort; as has appeared in ſome attempts of this kind.

In like manner, as little improvement is to be expected from water-mills with oblique vanes: for the power of the ſame ſection of a ſtream of water, is not greater when acting upon an oblique vane, than when acting upon a direct one: and any advantage that can be made by intercepting a greater ſection, which ſometimes may be done in the caſe of an open river, will be counterballanced by the ſuperior reſiſtance, that ſuch vanes would meet with by moving at right angles to the current: whereas the common floats always move with the water nearly in the ſame direction.

Here

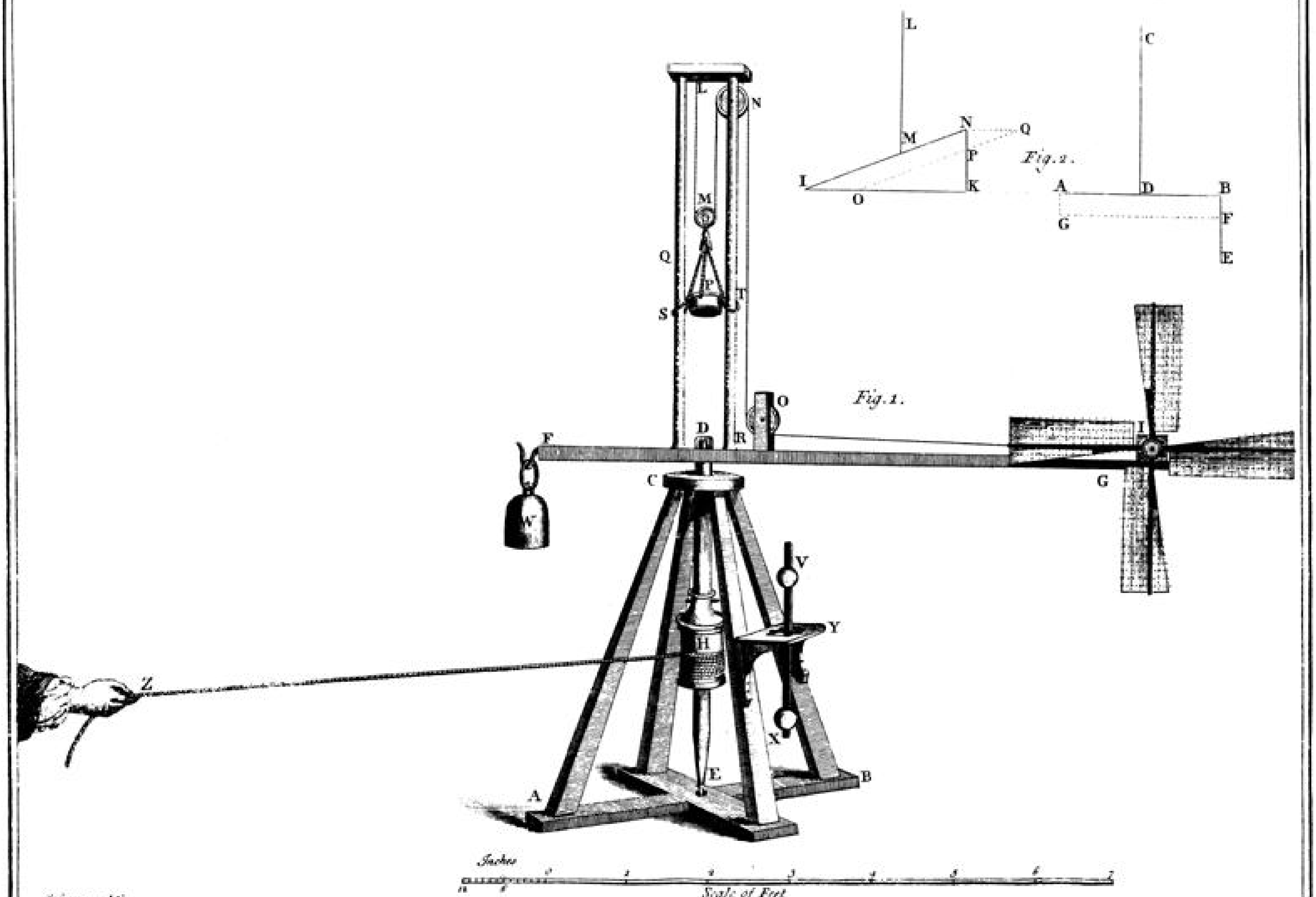
Here it may reaſonably be aſked, that ſince our geometrical demonſtration is general, and proves, that one angle of obliquity is as good as another; why in our experiments it appears, that there is a certain angle which is to be preferred to all the reſt? It is to be obſerved, that if the breadth of the ſail I N is given, the greater the angle K I N, and the leſs will be the baſe I K: that is, the ſection of wind interſected, will be leſs: on the other hand, the more acute the angle K I N, the leſs will be the perpendicular K N: that is, the impulſe of the wind, in the direction I K being leſs, and the velocity of the ſail greater; the reſiſtance of the medium will be greater alſo. Hence therefore, as there is a diminution of the ſection of the wind intercepted on one hand, and an increaſe of reſiſtance on the other, there is ſome angle, where the diſadvantage ariſing from theſe cauſes upon the whole is the leaſt of all; but as the diſadvantage ariſing from reſiſtance is more of a phyſical than geometrical conſideration, the true angle will beſt be aſſigned by experiment.

S C H O L I U M.

In trying the experiments contained in Tab. III. and IV. the different ſpecific gravity of the air, which is undoubtedly different at different times, will cauſe a difference in the load, proportional to the difference of its ſpecific gravity, tho' its velocity remains the ſame; and a variation of ſpecific gravity may ariſe not only from a variation of the weight of the whole column, but alſo by the difference of heat of the air concerned in the experiment, and poſſibly of other cauſes; yet the irregularities that might ariſe from a

dif-

difference of ſpecific gravity were thought to be too ſmall to be perceivable, till after the principal experiments were made, and their effects compared; from which, as well as ſucceeding experiments, thoſe variations were found to be capable of producing a ſenſible, tho' no very conſiderable effect: however, as all the experiments were tried in the ſummer ſeaſon, in the day-time, and under cover; we may ſuppoſe that the principal ſource of error would ariſe from the different weight of the column of the atmoſphere at different times; but as this ſeldom varies above $\frac{1}{17}$ part of the whole, we may conclude, that tho' many of the irregularities contained in the experiments referred to in the foregoing eſſay, might ariſe from this cauſe; yet as all the principal concluſions are drawn from the medium of a conſiderable number, many whereof were made at different times, it is preſumed that they will nearly agree with the truth, and be altogether ſufficient for regulating the practical conſtruction of thoſe kind of machines, for which uſe they were principally intended.



J. Smeaton delin.

J. Mynde sc.